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FARHAD DARUWALA
FRANK T. DENTON
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Frank Denton is a QSEP Research Associates and faculty members in the Department of Economics, McMaster University.

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Farhad Daruwala, Frank T. Denton and Dean C. Mountain

McMaster University

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Abstract

We consider optional TOU (time-of-use) pricing for residential consumers as an alternative to a single TOU or flat rate structure offered by a publicly regulated electricity supplier. A general equilibrium model is developed and used to explore and quantify the effects of optional pricing on welfare, consumption, and production costs. The model assumes that households can be classified into internally homogeneous groups with differing utility functions, incomes, demand elasticities, and committed consumption requirements. Substitution for electricity among TOU periods and between electricity and other goods is allowed for in the model on the demand side, and differing TOU-specific marginal costs on the supply side. The supplier in the model offers to each household a menu of possible rate sets obtained by maximizing a collective welfare function subject to three types of restriction: *Pareto efficiency* (no household is worse off under the proposed pricing scheme than under the current pricing scheme); *incentive compatibility* (every household weakly prefers its set of rates to the sets chosen by other households); *breakeven supplier revenue* (aggregate revenue must equal aggregate cost). The model is calibrated realistically with three household groups and three distinct TOU costing periods, and used in a series of simulation experiments, including experiments with alternative demand elasticities and marginal cost parameters. The use of optional pricing is shown to increase overall consumer welfare and reduce average production cost. However, the distribution of welfare effects can be uneven, with the highest income group dominating the market to the relative disadvantage of the lowest group. To deal with that situation an alternative strategy with a targeted rate structure for the lowest income group is proposed, corresponding to a modified version of the model specified in which some incentive compatibility restrictions are relaxed. Simulations show that the strategy can be effective in bringing about a more equitable distribution of welfare gains while still maintaining optional TOU pricing.

Keywords: Optional Differentiated Time-of-Use Rates, Pareto Efficiency, Incentive Compatibility, Welfare Benefits, General Equilibrium, Electricity Utility, Consumer Demand

JEL Classification: D11, D12, D82, Q41, D58, D61, L94

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I. INTRODUCTION

The literature on residential electricity pricing has concentrated, traditionally, on mandatory tariffs - one price structure for all. Public utilities and regulators, in setting rates, typically consider only a single representative consumer, and when there are discussions of fairness and equity they are carried out under the assumption that the rates will be imposed without any consumer choice. However, if we relax the mandatory constraint to allow for optional self-selected rates the efficiency and optimality implications can be quite different.

The obvious concern about optional time-of-use (TOU) rates is that customers will act as free riders in choosing a pricing option that lowers their bills without a similar reduction of production costs. On the other side of the argument, though, TOU rates that are optional (but realistically constrained) provide opportunities to those with the greatest ability to shift consumption away from peak demand periods, and thereby reduce average costs (e.g., see Caves, Christensen and Kuester, 1989). Willig's (1978) seminal article showed how optional tariffs in the context of a profit maximizing firm can improve the welfare of both consumers and the firm. An implicit assumption in that article and subsequent discussion was that demands by consumers are independent. On the other hand, Panzar and Sidak (2006) considered a firm with breakeven constraints where there are interdependent demands among consumers (e.g., purchases of one consumer group are influenced by the purchases of another); they noted that welfare effects from implementing optional tariffs in such a case may or may not be positive. Related papers by Rohlfs (1974), Katz, and Shapiro (1986), and Farrell and Saloner (1985), emphasized the positive externalities of interdependent demands. A theoretical model of optional tariff setting proposed by Rasanen, Ruusunen, and Hamalainen (1997), with illustrative application, was an important contribution to the literature. Our overall approach and model design in the present paper differ from theirs but we share their view as to the importance of exploring optional consumer choice in rate setting.

The challenge in setting a rate schedule with optional choice would be to design the optional price sets so as to recognize differences among households and maximize potential welfare and efficiency without sacrificing revenue recovery of costs.[1] It is clear that this would have to be accomplished in a general equilibrium setting where costs would be matched with revenues and tariff structures would fully take into account inter-period substitution of electricity usage, and substitutions with respect to non-electricity goods as well, for different types of consumers. We shall call a tariff-setting procedure designed in this context to allow consumer choice of TOU rates *differentiated TOU pricing*.

[1] Generally, as noted by Joskow (2012), most studies focus on average or aggregate demand responsiveness rather than considering heterogeneity across customers.

The replacement of flat (non-time-differentiated) electricity pricing with time-of-use or peak load pricing, aligned with marginal costs, has a long history in both the theoretical literature (Steiner, 1957, Boiteux, 1964a,b, Turvey, 1968, and Kahn, 1970) and the experimental literature (e.g., Mitchell and Acton, 1980, Caves, Christensen, and Herriges, 1984, Aigner, 1985, Mountain and Lawson, 1992, Ham, Mountain, and Chan, 1997, Wolak, 2006, Faruqui and Sergici, 2011). On the practical implementation front, however, as observed by Joskow and Wolfram (2012), very few utilities have actually moved to residential TOU rates in the United States; 98 percent of all U.S. residential customers have been on flat rates, as of recent years (Federal Energy Regulatory Commission, 2016, U.S. Energy Information Administration, 2016). In Canada, 67 percent of residential electricity customers are on flat rates, the remainder (all in Ontario) being on mandatory time-of use rates (Independent Electricity System Operator, 2012, Statistics Canada, 2011 for household data). (A few utilities in North America do offer a limited form of selected time-of-use rates, examples being in Arizona [Salt River Project, 2016] and Oklahoma [Oklahoma Gas & Electric, 2016].)

With the above as background we explore, in this paper, the impacts of optional differentiated household rates on consumer welfare benefits, costs, and related variables in a single-supplier general equilibrium model. The supplier is responsible to a public regulatory agency. It operates initially under either a flat rate pricing regime or a regime with TOU rates that are the same for all households – shared TOU prices, as we shall say – but it is about to shift to the new optional TOU price structure regime. Different groups of households are identified and each household in a group (with assumed homogeneous membership) will be allowed to choose a price structure from those offered by the supplier that maximizes its utility function. The aim of the supplier (under the supervision of the regulatory agency) is to offer to each household a menu of optional TOU price structures that collectively maximize an overall welfare function subject to restrictions ensuring Pareto efficiency (no household to have a rate structure that makes it worse off than before), incentive compatibility (no household to prefer any other available price structure to its own), and the further restriction that the supplier's revenue must be equal to its costs. (The restrictions in the model are consistent with the tariff design principles proposed by Wilson, [1993], and articles by Akerlof [1970], Spence [1973], and Rothschild and Stiglitz [1976], that emphasize the role of self-selection in achieving efficiency through the differentiation of prices for heterogeneous populations of customers.) The supplier (or regulator) in the model knows the overall distribution of demand preferences and can identify specific groups of households with particular preferences, but it cannot determine ex ante to which group an individual household belongs. The supplier must therefore offer a full menu of specific rate structures to every household. Households will then reveal their identities (group memberships) by the selections they make from the menu. The overall optimal solution to the model is a Nash equilibrium.

We calibrate the model and use it in a series of exploratory simulations. The simulations are designed to investigate the effects of alternative pricing schemes on prices, welfare, consumption, costs, and the distribution of output among higher and lower cost production periods. In particular, the simulations explore the effects of introducing differentiated TOU pricing when it replaces either a flat pricing or shared TOU pricing scheme. On the consumer side of the model, demand elasticities play a major role in determining the market equilibrium state under a differentiated pricing scheme; on the supply side, relative marginal costs in peak and other periods play a major role. We repeat the simulations under alternative specifications of these two sets of parameters to see how the results are affected.

Some households have much less discretion than others in their ability to take advantage of the availability of alternative price structures by changing their time patterns of consumption; their ratios of committed to total consumption of electricity in different TOU periods may be relatively high, a characteristic generally associated with lower incomes. With that in mind we introduce a modification of the model, as a possible policy supplement, by incorporating a targeted rate structure that is of benefit to low income households but is not available to households with higher incomes. Of special interest in these new simulations are the effects on welfare of the households that are targeted for the special treatment.

II. THE MODEL IN GENERAL FORM

Assume a single, non-profit supplier of electricity serving a population of N households. The population is divided into K internally homogeneous groups, each with a common set of relevant characteristics. The groups have populations n_k, k = 1,…,K. Initially electricity pricing is the same for all households but the supplier proposes now to introduce a new system of time-of-use (TOU) pricing. There are to be different rates for j = 1, …,J different periods of the day or week but the distinctive feature of the model is that the TOU rates need not be the same for all population groups. Recognizing the differences in group characteristics, the electricity supplier (or a regulatory agency to which it is responsible) seeks to set the TOU rates for each group so as to maximize the collective welfare of its customer population, subject to the requirement that overall cost must equal revenue. To make the pricing system work it must ensure that the new pricing system is *Pareto efficient*: no household is to be worse off (have a lower utility level) than under the old system. It must also ensure *incentive compatibility*: households in each group must prefer the set of rates that they choose to the sets chosen by households in each of the other groups. (Incentive compatibility ensures truth telling; that is, no group has an incentive to pretend to have another group's preferences. Consequently, as shown later, this leads to a Nash equilibrium.)

It is convenient to imagine a representative household for each group, household k, and to think of the TOU periods as representing J different electricity commodities. Household k's optimization problem is that of choosing a quantity demand vector q_k = $(q_{k0}, q_{k1},…,q_{kJ})'$, given (a) a set of nondiscretionary (committed) consumption requirements γ_k= $(\gamma_{k0}, \gamma_{k1}, \dots, \gamma_{kJ})'$, (b) a price vector p_k = $(p_{k0}, p_{k1},…,p_{kJ})'$, and (c) after-tax income y_k and discretionary income x_k = y_k - $p_k'\gamma_k$ (assumed by convention in the literature on demand analysis to be equal to discretionary expenditure).

The elements $p_{k1},…,p_{kJ}$ of the price vector are the TOU electricity prices assigned by the supplier to household k, p_{k0} is the price of all other goods, and the elements of q_k are the corresponding quantities. (To simplify presentation, without losing the essence of the discussion, we abstract from tariffs with a fixed part component; all electricity prices are volumetric.) The other goods price is taken as fixed and the same for all household groups: p_{k0} = p_0. The supplier's optimization problem is to choose the group-specific TOU electricity prices so as to maximize the collective welfare of the customer population, recognizing the revenue/cost, Pareto efficiency, and incentive compatibility restrictions. The model is of the following general form.

Household utility

Representative household k (for which discretionary consumption provides positive utility) has indirect utility v_k, a function of p_k and x_k:

$$v_k = v_k(x_k, p_k) \qquad (k = 1,\ldots,K)$$

Household demand

Corresponding to v_k (via Roy's identity) are household k's demand functions for commodities $j = 0,1,\ldots,J$:

$$q_{kj} = q_{kj}(x_k, p_k, \gamma_{kj}) \qquad (k = 1,\ldots,K)$$

Supplier output

The quantity of electricity supplied in TOU period j is equal to the population-weighted sum of the household demands in that period and the total quantity of electricity supplied is the sum of the period quantities:

$$Q_j = \sum_k n_k q_{kj} \qquad (j = 1,\ldots,J)$$

$$Q = \sum_j Q_j$$

Supplier cost

Electricity is produced with increasing marginal cost in every TOU period. The cost function in period j is $c_j(Q_j)$, yielding total cost C_j in that period and aggregate cost C, the sum over all periods:

$$C_j = c_j(Q_j) \qquad dC_j/dQ_j > 0, \;\; d^2C_j/dQ_j^2 > 0 \qquad (j = 1,\ldots,J)$$

$$C = \sum_j C_j$$

Note that the allowance for rising marginal costs contributes significantly to interdependence in the model; models of self-selection with constant marginal costs do not have this characteristic. (An example of the latter type of model is one used by Kolay and Shaffer [2003], in their examination of self-selection in the choice of bundling versus two-part tariffs, in the context of a profit maximizing monopolist.)

Supplier revenue

R_j, the supplier's revenue in TOU period j, is equal to electricity expenditures in that period summed over all household groups; aggregate revenue R is the sum of the period revenues:

$R_j = \sum_k n_k q_{kj} p_{kj}$ (j = 1,…,J)

$R = \sum_j R_j$

Welfare

The household indirect utility functions for the different groups are known to the electricity supplier. The supplier chooses group-specific TOU prices so as to maximize welfare W defined as a function of the groups' representative utility functions:

$$W = W(v_k(x_k, p_k);\ k = 1,\dots,K)$$

Pareto efficiency

In maximizing W, the supplier is constrained in its introduction of group-specific TOU prices so that every representative household's utility for its chosen set of prices is at least as great as it was before the introduction:

$$v_k(x_k, p_k) \geq v_k(x_k, r) \qquad (k = 1, \dots, K)$$

where r (the same for all groups) is the vector of prices prevailing before the switch to the new prices. (The price of other goods is unchanged, the same before and after: $r_{k0} = p_0$ for all k.)

Incentive compatibility

In maximizing W, the supplier is constrained so that every representative household prefers the set of electricity prices it chooses to the sets of prices chosen by the other representative households, or is indifferent, as represented by the inequalities

$$v_k(x_k, p_k) \geq v_k(x_k, p_s) \qquad (k, s = 1,\dots,K; \neq k)$$

(The price of other goods is a common element in the p_k and p_s vectors.)

Revenue/cost breakeven restriction

Revenue must equal cost:

$$R = C$$

(We use this form of restriction in our model but a modified version that allows a mark-up rate ξ over cost could easily be incorporated: $R = (1 + \xi)C$.) This is the last of the three explicit restrictions on the maximization of welfare in the model: Pareto efficiency, incentive compatibility, and cost equals revenue. An implicit fourth restriction is that customers must purchase some electricity in each TOU period, as represented by the nondiscretionary consumption requirements; they are not allowed to "opt out" of the supplier's pricing system by nonparticipation. These four restrictions together are consistent with the four tariff design principles proposed by R. D. Wilson (1993).

The constrained optimization process under *differentiated TOU pricing* described above can be stated more compactly as follows: the supplier maximizes the welfare of its customers by solving

$$\max_{p_1,p_2,\ldots,p_K} W(v_1(x_1,p_1), v_2(x_2,p_2), \ldots, v_K(x_K,p_K))$$

subject to:

(i) $v_k(x_k,p_k) \geq v_k(x_k,r)$ (k = 1, …,K), r being the common set of prices prevailing previously; (ii) $v_k(x_k, p_k) \geq v_k(x_k, p_s)$ (k, s = 1,…,K; $s \neq k$); and (iii) $R = C$.

Alternatively, for comparison, under the more restricted *shared TOU pricing*, with common price vector $p = (p_0,p_1,\ldots,p_J)'$, the supplier would solve

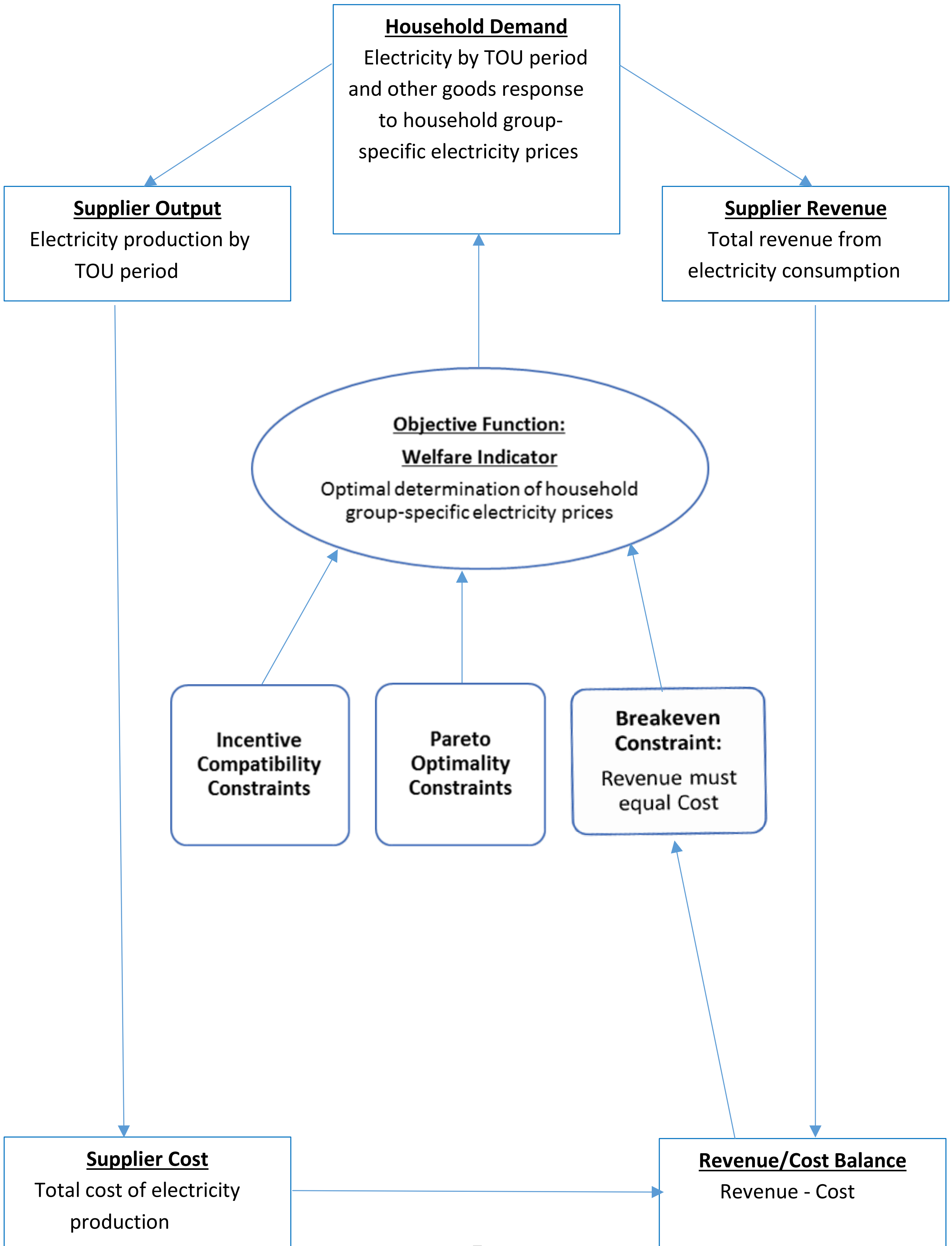
$$\max_{p} W(v_1(x_1,p), v_2(x_2,p), \ldots, v_K(x_K,p))$$

subject to (i) $v_k(x_k,p) \geq v_k(x_k,r)$ (k = 1, …, K); and (ii) $R = C$.

III. A SCHEMATIC REPRESENTATION

The general equilibrium model just outlined and its interactions are summarized in Figure 1 in order to provide a convenient overview. Starting at the top of the figure, households (of different types) generate the demand for goods, given a set of prices, and in particular the demand for electricity in different TOU periods. The demand for electricity then determines the output of the electricity supplier, by TOU period, on the left of the figure, and (given marginal cost parameters in conjunction with the TOU distribution) the total supplier cost, in the lower left corner. Household demand also determines the electricity supplier's total revenue, and cost and revenue come together in the lower right corner to determine the balance. But the balance must be checked against the breakeven constraint, which requires it to be zero. The Breakeven constraint along with the two other constraints, Pareto optimality and incentive compatibility, then determine the set of feasible TOU electricity prices available for consideration in maximizing the objective (welfare) function, in the centre of the figure. The maximization process determines the menu of optimum prices that the electricity supplier will offer to the different types of households (knowing their utility functions, and thus the choices they will make). In final equilibrium the menu of electricity prices offered to households by the supplier will be identical to the set of prices that determined household electricity demand in the beginning, and thus initiated the model process, as we have described it.

Figure 1: Diagrammatic Summary of the Model



IV. A DETAILED SPECIFICATION

The general form of the model allows alternative specifications for implementation. We adopt the following to illustrate its applicability.

As before, the electricity supplier sets out to introduce a new system of time-of-use pricing to a customer population of N households divided into K = 3 groups, each group being internally homogeneous with population n_k ($k = 1,2,3$). The number of TOU periods is now set at $J = 3$: peak, mid-peak, and off-peak periods for each day or week in a given year (we abstract from seasonal variations). The number of commodities (J) is thus four, consisting of three electricity commodities (j = 1,2,3) and a composite all-other-goods commodity (j = 0). Each representative household k has a corresponding vector of committed (price-insensitive) consumption γ_k = $(\gamma_{k0},\gamma_{k1},\gamma_{k2},\gamma_{k3})'$, all elements nonnegative. With total after-tax income y_k and price vector p_k= $(p_{k0},p_{k1},p_{k2},p_{k3})'$ the household has discretionary income (expenditure) $x_k = y_k - p_k'\gamma_k$, subject to which it chooses q_k so as to maximize its overall utility. For our illustrative specification, the corresponding maximum utility is represented by an indirect utility function v_k consistent with the linear expenditure system (LES) defined below:

$$v_k = x_k/P_k$$

where the overall price aggregator P_k for household k is given by

$$P_k = (\alpha_k(\bar{p}_k)^{\theta_k} + (1\text{-}\alpha_k)(p_0)^{\theta_k})^{1/\theta_k}$$

and its component electricity price aggregator by

$$\bar{p}_k = \prod_{j=1}^{3}(p_{kj})^{\beta_{kj}} \quad (\beta_{k1} + \beta_{k2} + \beta_{k3} = 1).$$

(See Keller [1976] for discussion of a nested quasi-homothetic CES formulation such as this one allowing for committed expenditures.)

This is consistent with the separability of electricity and other goods, which implies a two-stage budgeting process in which the first stage choice of aggregate consumption of electricity and other goods is a function of group price indexes (P_k and p_0) and the second stage choice of group- and time-specific quantities is a function of time-specific electricity prices and aggregate electricity expenditure. In consequence, the household allocates xe_k to electricity, in total, and xo_k to other goods, as follows:

$$xe_k = \alpha_k x_k(\bar{p}_k/P_k)^{\theta_k} \qquad xo_k = (1\text{-}\alpha_k)x_k(p_0/P_k)^{\theta_k} \qquad (xe_k + xo_k = x_k)$$

Correspondingly, the quantities demanded are generated for electricity by an LES (via Roy's identity) as

$$q_{kj} = \gamma_{kj} + \beta_{kj}(xe_k/p_{kj}) \quad (j\text{=}1,2,3)$$

and the quantity of other goods demanded can be calculated from $q_{k0} = xo_k/p_0$. (Note that the $\alpha, \beta,$ and θ coefficients can be derived from underlying demand elasticities, which is the way we calculate them in calibrating the model – see below. They can be viewed as taste parameters. Taking an approach different from that of Rasanen, Ruusunen, and Hamalainen [1997], we do not constrain taste to be represented by a single parameter on a continuum.)

The separability of electricity and other goods in the overall household utility function makes it possible to define an electricity-specific indirect utility function of the form

$$ve_k = xe_k/\bar{p}_k$$

thus allowing the utility effects of changes in electricity prices to be isolated. ve_k is defined as the indirectly derived utility from the consumption of electricity that is implied by the indirect overall utility function v_k. (See Appendix A for proof.)

The welfare function that the electricity supplier seeks to maximize is in effect a particular form of CES aggregator of the overall household utilities over the K household groups. For convenience of interpretation and application though we employ a money metric rather than use the utilities themselves (see Mackenzie, 1983), and write the function as

$$W = (\sum_{k=1}^{K} n_k (M_k/\bar{M}_k)^{\epsilon})^{1/\epsilon}$$

$M_k = B_k v_k$ and $\bar{M}_k = B_k \bar{v}_k$, where $\bar{v}_k$ is the overall utility level calculated *before* the change in the electricity pricing system. B_k corresponds exactly with P_k but is calculated with pre-change electricity prices. (All households face the same prices before the change; however, B carries a k subscript because households have different demand parameters and consequently weight the individual pre-change prices differently in the price aggregator, even though the prices themselves are the same for all households.) M_k is interpreted as the minimum total discretionary expenditure required to achieve overall utility level v_k with prices held at their initial levels and $\bar{M}_k$ is defined correspondingly with respect to $\bar{v}_k$. (This particular money metric formulation is consistent with an Allen quantity index; see Allen, 1949.) M_k and $\bar{M}_k$ are the *money metric equivalents* of the utilities associated, respectively, with the new and old prices, and their ratio is an indicator of the change in welfare resulting from the change in electricity prices. Since $M_k - \overline{M_k}$ is equal to equivalent variation, maximizing W is the same as maximizing a function of transformed equivalent variations. This is consistent with the maximization of social welfare for a public utility subject to a breakeven constraint, as discussed by Brown and Sibley (1986).

The separate availability of ve_k makes it possible to construct also a money metric utility measure ME_k specifically for electricity, at the level of household groups: $ME_k = Be_k ve_k$, where Be_k corresponds to B_k but relates only to electricity prices. ME_k is interpreted in a fashion analogous to M_k: it is the minimum discretionary expenditure on electricity that would have been required to achieve electricity utility level ve_k, with all electricity prices held at their initial levels. An equivalent household money metric measure $\overline{ME}_k$ can be calculated for the utility level that prevailed before the price changes, $\overline{ve}_k$, by replacing ve_k with $\overline{ve}_k$ in the above definition. The difference, ME_k - $\overline{ME}_k = Be_k(ve_k$-

$\overline{ve}_k$), is the household's *equivalent variation with respect to electricity*, which we label EVE_k, and the *proportionate* change is $EVE_k / \overline{ME}_k$. Corresponding aggregate money metric measures can be constructed as the population-weighted sums over household groups: $ME = \sum_k n_k ME_k$, $\overline{ME}$ = $\sum_k n_k \overline{ME}_k$, and EVE = $\sum_k n_k EVE_k$, with overall proportionate change $EVE/\overline{ME}$.

The electricity supplier's choice of the welfare-maximizing, group-specific prices is subject to the Pareto efficiency and incentive compatibility restrictions described in the previous section and we need not repeat the equations representing those restrictions. Also the same as in the previous section are the equations for supplier output (Q), cost (C), revenue (R), and the revenue/cost breakeven restriction (R=C), with the exception that the TOU cost functions are now given specific form. As in Rasanen, Ruusunen, and Hamalainen (1997), the particular TOU-specific cost functions that we specify are quadratic in output:

$$C_j = (\lambda_j + \phi_j Q_j)Q_j \qquad (j=1,2,3)$$

The j-subscripted parameters allow the functions to be different for different TOU periods to capture, in particular, differences in marginal costs among the peak, mid-peak, and off-peak periods. (Note that TOU differences in the electricity supplier's marginal costs may reflect the effects of patterns of use by industrial and commercial customers that are exogenous to the model, as well as the endogenous residential pattern determined within the model.)

V. ON THE NATURE OF THE EQUILIBRIUM

Equilibrium is established for the model by maximizing W = $(\sum_{k=1}^{K} n_k (M_k/\overline{M}_k)^{\epsilon})^{1/\epsilon}$ (k = 1,…,K) subject to the Pareto optimality, incentive compatibility, and revenue-equals-cost restrictions. (We write k = 1,…,K rather than k = 1,2,3 since the argument here generalizes to any number of households.) There are two characteristics of the equilibrium solution worth noting. The first is that it is a Nash equilibrium; the second is that it is stable under re-optimization.

Nash equilibrium: W is an increasing function of $(M_k/\overline{M}_k)$ = $(v_k/\overline{v}_k)$. If a higher level of utility v_k and hence M_k could be reached for any household k without violating the incentive compatibility restrictions on the other households, it would already have been incorporated into the maximization solution for W. Thus given the prices chosen by the other households no household could improve its own utility level by making a different choice of prices resulting in a Nash equilibrium.

Stability under re-optimization: Suppose the original maximization of W starts from some arbitrary initial state and yields an optimal set of utilities $\hat{v}_k$, prices $\hat{p}_k$, corresponding money metric equivalents $\widehat{M}_k$, for all k, and a maximum welfare level $\widehat{W}$. Now suppose we take these values as defining a new initial state and carry out a new round of optimization - we sequentially *re-optimize*. We would now optimize by maximizing $W = (\sum_{k=1}^{K} n_k (M_k/\widehat{M}_k)^{\epsilon})^{1/\epsilon}$, obtaining in the solution new utility levels, prices, and money metric equivalents v_k^*, p_k^*, M_k^*, and a new index of maximum welfare W^*. The solution is constrained by the same restrictions as before, including in particular the Pareto optimality restrictions which can be written as $v_k^*/\hat{v}_k$ = $M_k^*/\widehat{M}_k$ ≥ 1 for all k. A value W^* > 1 would indicate an increase in welfare as a result of the reoptimization. The Pareto restrictions make a value W^* < 1 impossible and it can be shown that these restrictions also preclude the existence of any W^* > 1, and so require M_k^* = $\widehat{M}_k$ for all k and W^* = 1. The proof is by contradiction: Suppose there does exist a W^* > 1; that means that there must exist an M_k^* for at least one k for which $M_k^*/\widehat{M}_k$ > 1. But that is impossible since with v_k^* ≥ $\hat{v}_k$ ≥ $\overline{v}_k$ it would have been taken into account in the original optimization and $\widehat{W}$ would have been greater. Reoptimization thus leaves the equilibrium solution unchanged.

VI. CALIBRATION

We turn now to a calibration of the model for use in a series of simulation experiments and that requires assumptions about the supply and demand characteristics of the electricity market. The basic inputs and derived parameter values for the initial simulations are shown in Table 1. The treatment here is in summary form; additional details are provided in Appendix B.

Starting with the demand side of the model the inputs are of five kinds, as shown in the table. (1) After-tax average incomes for the three types of households: y_k, k = 1,2,3, representing households with low, medium, and high income levels, and correspondingly high, medium, and low demand price elasticities. (2) Committed consumption of non-electricity goods for each household type (γ_{k0}). (3) Aggregate consumption of electricity (q_k) for each household type in the reference state (flat rates). (4) Demand (own) price elasticities for aggregate electricity (η_k) and for electricity in each TOU period (η_{kj}, j = 1,2,3, relating to peak, mid-peak, and off-peak periods). (5) Two selected characteristics of electricity consumption in the reference state: $R21_k$, the ratio of mid-peak to peak consumption for household k, and R_{31}, the ratio of off-peak to peak consumption. (These ratios, like aggregate electricity consumption, are necessary to obtain parameter values and a model solution for the initial state. They change, of course, when variable prices are introduced.) The γ_k and γ_{k0} values are based on data from the Survey of Household Spending, Statistics Canada, 2010. The q_k, $R21_k$, and $R31_k$values are based on sample data sets from households served by Hydro One, the largest electricity supplier in Ontario, Canada. The η_k and η_{kj} values are based on a survey of numerous articles (Caves and Christensen, 1980, Parks and Weitzel, 1981, Caves, Christensen, Schoech, and Hendricks, 1984, Mountain, 1993, Mountain and Lawson, 1992, 1995, Faruqui and Sergici, 2010, and others). Furthermore, we assume that $|\eta_k|$ and $|\eta_{kj}|$ increase as discretionary income declines (see Wolak, 2010). The inputs just described provide the basis for calculating all of the explicit standard parameter values in the equations that represent the demand side of the model. Specifically, they allow the calculation of β_{kj}, γ_{kj}, k = 1,2,3, j = 1,2,3, and α_k, θ_k, k = 1,2,3 (see Appendix B). (Note that the parameter ϵ in the welfare function is set to 1 in all of the simulations that we report and that the household groups are all of the same size: n_k = N/3, k = 1, 2, 3.)

On the supply side the inputs relate to an average week in the year. They include, for each TOU period j, assumed lower and upper bounds on the marginal cost of electricity production ($\underline{mc}_j, \overline{mc}_j$) and the number of hours in the period (HR_j). The $\underline{mc}_j$ and $\overline{mc}_j$ values are based on ranges of electricity costs

Table 1. Input Reference Values and Derived Parameters

Input Values: Demand	Household (k)		
	Low Discretionary Electricity Expenditure ($k = 1$)	Medium Discretionary Electricity Expenditure ($k = 2$)	High Discretionary Electricity Expenditure ($k = 3$)
y_k ($)	28,350	54,600	100,750
γ_{k0} ($)	12,400	19,650	31,000
q_k (kWh)	3,680	7,090	13,100
η_k	-0.1870	-0.1350	-0.0900
η_{k1}	-0.2145	-0.1925	-0.1595
η_{k2}	-0.1600	-0.1400	-0.1100
η_{k3}	-0.1170	-0.0900	-0.0720
$R21_k$	0.77	0.87	0.95
$R31_k$	1.60	2.0	2.25

Input Values: Supply	TOU Period (j)		
	Peak ($j = 1$)	Mid-Peak ($j = 2$)	Off-Peak ($j = 3$)
$\underline{mc}_j$ ($/kWh)	0.15	0.10	0.05
$\overline{mc}_j$ ($/kWh)	0.35	0.25	0.12
HR_j (hrs/week)	30	30	108

Derived Parameters: Demand	Household (k)		
	Low Discretionary Electricity Expenditure ($k = 1$)	Medium Discretionary Electricity Expenditure ($k = 2$)	High Discretionary Electricity Expenditure ($k = 3$)
β_{k1}	0.2990	0.2152	0.2042
β_{k2}	0.1757	0.1516	0.1377
β_{k3}	0.5253	0.6332	0.6580
γ_{k1}(kWh)	758	1,459	2,660
γ_{k2} (kWh)	644	1,331	2,654
γ_{k3} (kWh)	1,160	2,565	5,540
α_k	0.00712	0.00504	0.00327
θ_k	0.3870	0.4507	0.4763

Derived Parameters: Supply	TOU Period (j)		
	Peak ($j = 1$)	Mid-Peak ($j = 2$)	Off-Peak ($j = 3$)
λ_j ($/kWh)	0.0833	0.0500	0.0267
ϕ_j	0.0213	0.0168	0.0141

in selected North American electrical utilities (including Ontario). The HR_j values reflect the current specification of hours in the peak, mid-peak, and off-peak TOU periods in Ontario. The two parameters of the cost function, λ and φ, are then calculated, as described in Appendix B.

VII. SIMULATIONS: TABLE STRUCTURE

The results of a series of simulations are provided in Tables 2, 3, and 4, each table being divided into two parts (2a, 2b, etc.). The simulations have two purposes: (1) to see how results under the differentiated TOU pricing scheme built into the model compare with results based on two common alternatives, flat pricing and shared TOU pricing; (2) to explore the sensitivity of the results to changes in two sets of key parameters, demand elasticities and marginal costs.

Assume an electricity provider operating with a flat pricing scheme but considering shifting to a shared TOU scheme (all households have the same TOU prices); the comparison is made in Table 2. Now assume that the same supplier, having made the initial change, is considering a further shift, from shared TOU to differentiated TOU pricing (household groups have different TOU prices); the shared vs. differentiated comparison is provided in Table 3. Finally, the effects had the supplier gone directly from flat to differentiated TOU pricing are provided in Table 4. In addition to results based on the standard parameter values in each table the (a) part of the table shows, for each of the pricing schemes being compared, how results are affected by lowering demand price elasticities or by raising them and the (b) part shows the effects of doing the same with cost parameters. Aside from these differences the tables are structured similarly. On the left side they show the three TOU prices for a representative household in each group (p_1, p_2, p_3), the changes in electricity welfare (group-specific and overall) resulting from the shift in pricing scheme ($EVE/\overline{ME}$, shown in percentage form), percentage changes in electricity supplier output (%ΔQ), total cost (%ΔC), and average cost (%Δ(C/Q)), all relative to pre-change levels, and the distribution of costs among the three TOU periods (C_1/Q, C_2/Q, C_3/Q, in percentage form). On the right side of each table they show the percentage changes in electricity consumption for each household group, by TOU period (%Δq_1, %Δq_2, %Δq_3) and in total (%Δq), and the TOU distribution of supplier output (Q_1/Q, Q_2/Q, Q_3/Q, again in percentage form).

Collectively, Tables 2, 3, and 4 provide a comprehensive picture of relative performance under the three alternative pricing schemes and in particular, for our purposes, a comparison of how the differentiated pricing scheme built into our model performs relative to the others. (For proper comparison, all three sets of prices are generated from the same basic model - the flat prices by adding to the model the restriction that all prices must be the same, the shared TOU prices by adding instead the restriction that all household groups must have the same set of TOU prices.) The tables also provide indications of how relative performance under each pricing scheme is affected by alternative assumptions about two fundamental characteristics of the electricity market: price elasticities on the demand side, cost functions on the supply side.

Table 2a. Moving from Flat Pricing to Shared TOU Pricing: The Effects of Alternative Demand Price Elasticities on Prices, Welfare, Costs, Consumption, and Output

Prices and Costs	Reference	Standard Price Elasticities	Lower Price Elasticities	Higher Price Elasticities
Household 1				
p_1	1.000	1.189	1.103	1.283
p_2	1.000	1.131	1.119	1.119
p_3	1.000	0.828	0.882	0.784
$EVE/\overline{ME}$ (%)	0.0	1.6	1.0	2.0
Household 2				
p_1	1.000	1.189	1.103	1.283
p_2	1.000	1.131	1.119	1.119
p_3	1.000	0.828	0.882	0.784
$EVE/\overline{ME}$ (%)	0.0	3.6	2.3	4.7
Household 3				
p_1	1.000	1.189	1.103	1.283
p_2	1.000	1.131	1.119	1.119
p_3	1.000	0.828	0.882	0.784
$EVE/\overline{ME}$ (%)	0.0	3.8	2.5	5.0
Overall $EVE/\overline{ME}$ (%)	0.0	3.2	2.2	4.2
Production and Costs				
%ΔQ	-	0.99	0.54	1.52
%ΔC	-	-0.83	-0.56	-0.99
%Δ(C/Q)	-	-1.80	-1.11	-2.47
C_1/C (%)	43.5	41.6	42.5	40.5
C_2/C (%)	25.3	24.8	24.9	24.7
C_3/C (%)	31.2	33.7	32.6	34.8

Consumption and Output	Reference	Standard Price Elasticities	Lower Price Elasticities	Higher Price Elasticities
Household 1				
$\%\Delta q_1$	-	-5.12	-2.59	-8.13
$\%\Delta q_2$	-	-2.91	-2.23	-3.16
$\%\Delta q_3$	-	6.56	3.62	9.99
$\%\Delta q$	-	0.93	0.44	1.61
Household 2				
$\%\Delta q_1$	-	-3.72	-1.91	-5.85
$\%\Delta q_2$	-	-2.32	-1.73	-2.74
$\%\Delta q_3$	-	5.20	2.89	7.87
$\%\Delta q$	-	1.20	0.61	1.96
Household 3				
$\%\Delta q_1$	-	-2.75	-1.67	-3.14
$\%\Delta q_2$	-	-1.52	-1.32	-1.51
$\%\Delta q_3$	-	3.52	2.30	4.63
$\%\Delta q$	-	0.89	0.54	1.25
Overall TOU Shares				
Q_1/Q (%)	25.3	24.2	24.7	23.6
Q_2/Q (%)	22.6	22.0	22.1	21.8
Q_3/Q (%)	52.1	53.9	53.2	54.5

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q). The standard price elasticities for households 1 and 2 are multiplied by 0.85 under "Lower Price Elasticities" and by 1.15 under "Higher Price Elasticities".

Table 2b. Moving from Flat Pricing to Shared TOU Pricing: The Effects of Alternative Peak and Mid-Peak Cost Parameters on Prices, Welfare, Costs, Consumption, and Output

Prices and Costs	Reference	Standard Costs	Lower Costs	Higher Costs
Household 1				
p_1	1.000	1.189	1.049	1.320
p_2	1.000	1.131	1.042	1.161
p_3	1.000	0.828	0.950	0.745
$EVE/\overline{ME}$ (%)	0.0	1.6	0.3	2.8
Household 2				
p_1	1.000	1.189	1.049	1.320
p_2	1.000	1.131	1.042	1.161
p_3	1.000	0.828	0.950	0.745
$EVE/\overline{ME}$ (%)	0.0	3.6	0.9	5.9
Household 3				
p_1	1.000	1.189	1.049	1.320
p_2	1.000	1.131	1.042	1.161
p_3	1.000	0.828	0.950	0.745
$EVE/\overline{ME}$ (%)	0.0	3.8	1.0	6.3
Overall $EVE/\overline{ME}$ (%)	0.0	3.2	0.8	5.4
Production and Costs				
%ΔQ	-	0.99	0.20	1.86
%ΔC	-	-0.83	-0.27	-1.16
%Δ(C/Q)	-	-1.80	-0.47	-2.96
C_1/C (%)	43.5	41.6	41.4	41.5
C_2/C (%)	25.3	24.8	24.5	25,0
C_3/C (%)	31.2	33.7	34.1	33.4

Consumption and Output	Reference	Standard Costs	Lower Costs	Higher Costs
Household 1				
$\%\Delta q_1$	-	-5.12	-1.49	-7.83
$\%\Delta q_2$	-	-2.91	-0.99	-3.60
$\%\Delta q_3$	-	6.56	1.70	10.78
$\%\Delta q$	-	0.93	1.37	1.95
Household 2				
$\%\Delta q_1$	-	-3.72	-1.09	-5.65
$\%\Delta q_2$	-	-2.32	-0.78	-2.94
$\%\Delta q_3$	-	5.20	1.36	8.44
$\%\Delta q$	-	1.20	0.24	2.24
Household 3				
$\%\Delta q_1$	-	-2.75	-0.81	-4.16
$\%\Delta q_2$	-	-1.52	-0.51	-1.93
$\%\Delta q_3$	-	3.52	-0.92	5.70
$\%\Delta q$	-	0.89	0.19	1.62
Overall TOU Shares				
Q_1/Q (%)	25.3	24.2	25.0	23.5
Q_2/Q (%)	22.6	22.0	22.4	21.6
Q_3/Q (%)	52.1	53.9	52.6	54.8

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q). The standard cost parameters (λ, ϕ) for TOU periods 1 and 2 are multiplied by 0.85 under “Lower Costs” and by 1.15 under “Higher Costs”.

Table 3a. Moving from Shared TOU Pricing to Differentiated TOU Pricing: The Effects of Alternative Demand Elasticities on Prices, Welfare, Costs, Consumption, and Output

Prices and Costs	Reference	Standard Price Elasticities	Lower Price Elasticities	Higher Price Elasticities
Household 1				
p_1	1.189	1.171	1.157	1.221
p_2	1.131	1.030	1.034	1.038
p_3	0.828	0.876	0.883	0.846
$EVE/\overline{ME}$ (%)	0.0	-0.5	-0.6	-0.2
Household 2				
p_1	1.189	1.335	1.261	1.380
p_2	1.131	1.188	1.202	1.176
p_3	0.828	0.739	0.767	0.722
$EVE/\overline{ME}$ (%)	0.0	2.2	1.5	2.7
Household 3				
p_1	1.189	1.535	1.548	1.522
p_2	1.131	1.053	1.061	1.044
p_3	0.828	0.712	0.708	0.718
$EVE/\overline{ME}$ (%)	0.0	3.0	3.1	2.9
Overall $EVE/\overline{ME}$ (%)	0.0	2.0	1.8	2.1
Production and Costs				
%ΔQ	-	0.88	0.76	1.02
%ΔC	-	0.09	0.08	0.08
%Δ(C/Q)	-	-0.77	-0.67	-1.00
C_1/C (%)	41.6	40.2	40.6	39.7
C_2/C (%)	24.8	24.9	25.0	24.8
C_3/C (%)	33.7	34.9	34.4	35.5

Consumption and Output	Reference	Standard Price Elasticities	Lower Price Elasticities	Higher Price Elasticities
Household 1				
$\%\Delta q_1$	-	0.50	0.71	-0.76
$\%\Delta q_2$	-	2.12	1.75	2.23
$\%\Delta q_3$	-	-1.92	-1.89	-0.83
$\%\Delta q$	-	-0.36	-0.35	-0.14
Household 2				
$\%\Delta q_1$	-	-2.16	-1.00	-3.15
$\%\Delta q_2$	-	-0.95	-0.86	-0.99
$\%\Delta q_3$	-	3.37	1.92	4.65
$\%\Delta q$	-	1.07	0.59	1.53
Household 3				
$\%\Delta q_1$	-	-3.03	-3.11	-2.93
$\%\Delta q_2$	-	0.41	-0.33	0.50
$\%\Delta q_3$	-	3.13	3.28	2.95
$\%\Delta q$	-	1.11	1.16	1.06
Overall TOU Shares				
Q_1/Q (%)	24.2	23.5	23.7	23.2
Q_2/Q (%)	22.0	21.8	21.9	21.8
Q_3/Q (%)	53.9	54.7	54.4	55.0

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q). The standard price elasticities for households 1 and 2 are multiplied by 0.85 under "Lower Price Elasticities" and by 1.15 under "Higher Price Elasticities".

Table 3b. Moving from Shared TOU Pricing to Differentiated TOU Pricing: The Effects of Alternative Peak and Mid-Peak Cost Parameters on Prices, Welfare, Costs, Consumption, and Output

	Prices and Costs			
	Reference	Standard Costs	Lower Costs	Higher Costs
Household 1				
p_1	1.189	1.171	1.213	1.216
p_2	1.131	1.030	1.048	1.075
p_3	0.828	0.876	0.852	0.828
$EVE/\overline{ME}$ (%)	0.0	-0.5	-0.4	0.2
Household 2				
p_1	1.189	1.335	1.187	1.373
p_2	1.131	1.188	1.180	1.228
p_3	0.828	0.739	0.809	0.699
$EVE/\overline{ME}$ (%)	0.0	2.2	0.5	3.6
Household 3				
p_1	1.189	1.535	1.482	1.572
p_2	1.131	1.053	1.016	1.075
p_3	0.828	0.712	0.755	0.680
$EVE/\overline{ME}$ (%)	0.0	3.0	1.6	4.3
Overall $EVE/\overline{ME}$ (%)	0.0	2.0	0.8	3.1
Production and Costs				
%ΔQ	-	0.88	0.39	1.35
%ΔC	-	0.09	0.00	0.10
%Δ(C/Q)	-	-0.77	-0.38	-1.23
C_1/C (%)	41.6	40.2	39.2	41.0
C_2/C (%)	24.8	24.9	24.2	25.1
C_3/C (%)	33.7	34.9	36.6	33.9

	Consumption and Output			
	Reference	Standard Costs	Lower Costs	Higher Costs
Household 1				
$\%\Delta q_1$	-	0.50	-0.45	-0.62
$\%\Delta q_2$	-	2.12	1,72	1.08
$\%\Delta q_3$	-	-1.92	-0.94	-0.01
$\%\Delta q$	-	-0.36	-0.22	0.06
Household 2				
$\%\Delta q_1$	-	-2.16	-0.04	-2.74
$\%\Delta q_2$	-	-0.95	-0.66	-1.54
$\%\Delta q_3$	-	3.37	0.65	5.07
$\%\Delta q$	-	1.07	0.20	1.72
Household 3				
$\%\Delta q_1$	-	-3.03	-2.57	-3.34
$\%\Delta q_2$	-	0.41	0.88	0.12
$\%\Delta q_3$	-	3.13	1.92	4.11
$\%\Delta q$	-	1.11	0.66	1.52
Overall TOU Shares				
Q_1/Q (%)	24.2	23.5	23.8	23.2
Q_2/Q (%)	22.0	21.8	22.0	21.6
Q_3/Q (%)	53.9	54.7	54.3	55.2

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q). The standard cost parameters (λ, ϕ) for TOU periods 1 and 2 are multiplied by 0.85 under "Lower Costs" and by 1.15 under "Higher Costs".

Table 4a. Moving from Flat Pricing to Differentiated TOU Pricing: The Effects of Alternative Demand Elasticities on Prices, Welfare, Costs, Consumption, and Output

Prices and Costs	Reference	Standard Price Elasticities	Lower Price Elasticities	Higher Price Elasticities
Household 1				
p_1	1.000	1.171	1.110	1.221
p_2	1.000	1.031	1.011	1.038
p_3	1.000	0.876	0.918	0.845
$EVE/\overline{ME}$ (%)	0.0	1.1	0.7	1.3
Household 2				
p_1	1.000	1.335	1.282	1.380
p_2	1.000	1.188	1.195	1.176
p_3	1.000	0.739	0.768	0.722
$EVE/\overline{ME}$ (%)	0.0	5.8	4.2	6.4
Household 3				
p_1	1.000	1.535	1.550	1.522
p_2	1.000	1.053	1.062	1.044
p_3	1.000	0.712	0.707	0.718
$EVE/\overline{ME}$ (%)	0.0	7.0	7.1	6.9
Overall $EVE/\overline{ME}$ (%)	0.0	5.3	5.2	5.4
Production and Costs				
%ΔQ	-	1.87	1.67	2.09
%ΔC	-	-0.73	-0.73	-0.81
%Δ(C/Q)	-	-2.56	-2.26	-2.84
C_1/C (%)	43.5	40.2	40.7	39.7
C_2/C (%)	25.3	24.9	25.0	24.7
C_3/C (%)	31.2	34.9	34.4	35.5

Consumption and Output	Reference	Standard Price Elasticities	Lower Price Elasticities	Higher Price Elasticities
Household 1				
$\%\Delta q_1$	-	-4.65	-2.68	-6.60
$\%\Delta q_2$	-	-0.85	-0.31	-1.19
$\%\Delta q_3$	-	4.52	2.41	6.66
$\%\Delta q$	-	0.57	0.28	0.93
Household 2				
$\%\Delta q_1$	-	-5.81	-4.36	-7.29
$\%\Delta q_2$	-	-3.24	-2.76	-3.63
$\%\Delta q_3$	-	8.75	6.59	10.91
$\%\Delta q$	-	2.29	1.66	2.94
Household 3				
$\%\Delta q_1$	-	-5.69	-5.78	-5.60
$\%\Delta q_2$	-	-1.11	-1.20	-1.02
$\%\Delta q_3$	-	6.75	6.93	6.57
$\%\Delta q$	-	2.01	2.06	1.96
Overall TOU Shares				
Q_1/Q (%)	25.3	23.5	23.7	23.2
Q_2/Q (%)	22.6	21.8	21.9	21.7
Q_3/Q (%)	52.1	54.7	54.4	55.0

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q). The standard price elasticities for households 1 and 2 are multiplied by 0.85 under "Lower Price Elasticities" and by 1.15 under "Higher Price Elasticities".

Table 4b. Moving from Flat Pricing to Differentiated TOU Pricing: The Effects of Alternative Peak and Mid-Peak Cost Parameters on Prices, Welfare, Costs, Consumption, and Output

Prices and Costs	Reference	Standard Costs	Lower Costs	Higher Costs
Household 1				
p_1	1.000	1.171	1.116	1.216
p_2	1.000	1.031	0.975	1.075
p_3	1.000	0.876	0.935	0.828
$EVE/\overline{ME}$ (%)	0.0	1.1	0.4	1.7
Household 2				
p_1	1.000	1.335	1.285	1.373
p_2	1.000	1.188	1.132	1.228
p_3	1.000	0.739	0.792	0.699
$EVE/\overline{ME}$ (%)	0.0	5.8	4.2	7.3
Household 3				
p_1	1.000	1.535	1.486	1.572
p_2	1.000	1.053	1.022	1.075
p_3	1.000	0.712	0.755	0.680
$EVE/\overline{ME}$ (%)	0.0	7.0	5.5	8.3
Overall $EVE/\overline{ME}$ (%)	0.0	5.3	3.9	6.5
Production and Costs				
%ΔQ	-	1.87	1.35	2.35
%ΔC	-	-0.73	-0.49	-0.97
%Δ(C/Q)	-	-2.56	-1.81	-3.25
C_1/C (%)	43.5	40.2	39.2	41.0
C_2/C (%)	25.3	24.9	24.4	25.1
C_3/C (%)	31.2	34.9	36.4	33.9

Consumption and Output	Reference	Standard Costs	Lower Costs	Higher Costs
Household 1				
$\%\Delta q_1$	-	-4.65	-3.26	-5.71
$\%\Delta q_2$	-	-0.85	0.54	-1.86
$\%\Delta q_3$	-	4.52	2.23	6.55
$\%\Delta q$	-	0.57	0.22	0.99
Household 2				
$\%\Delta q_1$	-	-5.81	-5.05	-7.00
$\%\Delta q_2$	-	-3.24	-2.41	-3.82
$\%\Delta q_3$	-	8.75	6.61	10.54
$\%\Delta q$	-	2.29	1.57	2.94
Household 3				
$\%\Delta q_1$	-	-5.69	-5.27	-6.00
$\%\Delta q_2$	-	-1.11	-0.70	-1.40
$\%\Delta q_3$	-	6.75	5.52	7.77
$\%\Delta q$	-	2.01	1.55	2.42
Overall TOU Shares				
Q_1/Q (%)	25.3	23.5	23.8	23.2
Q_2/Q (%)	22.6	21.8	22.1	21.6
Q_3/Q (%)	52.1	54.7	54.1	55.2

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q). The standard cost parameters (λ, ϕ) for TOU periods 1 and 2 are multiplied by 0.85 under "Lower Costs" and by 1.15 under "Higher Costs".

VIII. SIMULATIONS: COMPARISONS OF PRICING SCHEMES

We begin by comparing the effects of alternative pricing schemes using the simulation results based on standard parameter values. There are two points to be noted with regard to the comparisons. The first is that, while price changes can be substantial (sometimes over 50 percent), consumption effects in moving from one scheme to another are generally of small order. Given the nature of the electricity "commodity" with its fixed utilization commitments and consequent relatively low overall demand elasticities one should not expect large increases or decreases in consumption when one price scheme is replaced by another. But even small consumption changes can have large absolute effects on aggregate welfare benefits and production costs at the margin, including the distribution of output among higher and lower cost TOU periods. (A small shift from peak period to mid-peak or off-peak consumption could have a much larger proportional effect on average cost.) The restrictions on "fairness" and "equity" as perceived by the customers of a supplier, and represented by the Pareto and incentive compatibility restrictions in the model (and by similar but less formal constraints on price setting in a real market), limit further the degree of variation that is possible. The second point to note is that the model is nonlinear and so the changes in welfare, consumption, and production in going from flat to shared TOU pricing, and then from shared to differentiated TOU pricing (Tables 2 and 3) need not add up to the changes that would be realized in going directly from flat to differentiated TOU pricing (Table 3). However, in fact the results in the tables are quite close to being additive. It is thus possible to think of the overall flat-to-differentiated changes as consisting roughly of the two components. We report *proportional* changes in welfare, consumption, and production in the tables, and those of course would not be additive in any event but the proportional changes are of small order, as noted, and thus they too are consistent with adding up as an approximation.

Turning to the substance of the tables, we note that under differentiated pricing the peak to off-peak price differentials are largest for households that have the highest discretionary expenditures (and lowest demand elasticities): the peak to off-peak ratios are 1.3, 1.8, and 2.2 for the low, medium, and high discretionary households, respectively, households 1, 2, and 3. (For shared TOU pricing the ratio is 1.4.) This result is consistent with the observation that when Ramsey pricing is used, price differentials are inversely proportional to consumers' demand price elasticities (see Baumol and Bradford, 1970).

All household groups show electricity welfare increases in going from flat to shared pricing (Table 2). Overall electricity welfare increases further in going from shared to differentiated pricing (Table 3) and household groups 2 and 3 participate in the increase, but not group 1, the group with the least discretionary electricity expenditure; that group loses some electricity welfare. This illustrates an important feature of the market under differentiated pricing. The model is a general equilibrium model, and so choices made by the biggest consumers of electricity (groups 2 and 3, but especially 3) have a disproportionately large effect on the market equilibrium that is established. To a considerable degree therefore household group 1, the poorest group, with the least discretionary leverage on the equilibrium outcome, is at the mercy of the other household groups, so to speak. (We consider below a possible modification of the model that could be used to offset this disadvantage for group 1.) In sum, then, differentiated pricing increases overall electricity welfare but has uneven distributional effects. In support of the earlier observation on approximate adding up, we note that flat to shared, shared to differentiated, and flat to differentiated pricing changes yield overall welfare increases of 3.2, 2.0, and 5.3 percent. These results may be compared with estimates of welfare benefits in going from flat to (shared) TOU pricing in the range of -3 to 8 percent made by Parks and Weitzel (1984). Also Caves et al. (1984) found the welfare gains for a similar change of pricing regime to be very small in the case of four major Illinois utilities: they obtained a present value (as of 1982) of $20.3 million, calculated over 10 years for about 4 million customers. Our estimate of a 3.2 percent increase in electricity welfare (EVE, equivalent variation) in going from flat to shared TOU pricing (Table 2a, with standard price elasticities) is equivalent to a present value (as of 2016) of $1.649 billion for 5 million Ontario households, calculated over 10 years; the 2.0 percent increase in going from shared to differentiated TOU pricing (Table 3a) is equivalent to $968 million; and the 5.3 percent increase in going from flat to differentiated pricing (Table 4a) is equivalent to $2.688 billion. ($EVE$ is calculated here based on a flat rate of $0.199 per kWh, the recent average price of a Hydro One residential customer in its medium density region.)

The same general pattern that holds for welfare in our calculations holds also for consumption. Consumption of electricity increases for all groups in going from flat to shared pricing but only for groups 2 and 3 in going from shared to differentiated pricing; consumption of group 1 declines somewhat in the latter transition. (The overall increases for the three transitions are 0.99, 0.88, and 1.87 percent.)

On the supply side, aggregate production (equals aggregate consumption) increases in going from flat to shared pricing and again from shared to differentiated pricing. Average cost decreases at both stages – by 1.8 percent in going from flat to shared TOU pricing and by 0.77 percent in going from shared to differentiated TOU pricing. It decreases by 2.56 percent in going from flat directly to shared TOU pricing. (The reduction of average cost is in fact a common feature of pricing scheme transitions in all of the simulation experiments, including the experiments with alternative elasticity and marginal cost parameters discussed below.) Total cost declines in going from flat to shared pricing but the increase in output offsets the reduction in average cost, resulting in a slight increase in total cost when differentiated pricing replaces shared pricing. The reductions in average costs come from the shifting of demand, especially from peak to the other periods. The peak TOU period's share of the total declines at each stage of transition, both for output and cost.

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IX. SIMULATIONS: ALTERNATIVE ELASTICITIES AND COST PARAMETERS

These simulations explore the sensitivity of the equilibrium results to changes in demand price elasticities and cost parameters. The effects of alternative elasticities are shown in the (a) parts of Tables 2, 3, and 4, the effects of alternative cost parameters are shown in the (b) parts. (Note that when elasticities or cost parameters are altered in an experiment the reference values – the values before the change in pricing regime – must be recalculated accordingly so that the two pricing schemes under comparison are operative with the same set of parameters. To save space the recalculated reference values are not shown in the tables; only the ones based on standard parameter values.) The elasticity experiments involve alternatively decreasing the elasticities of household groups 1 and 2 (the households with the largest elasticities) by a factor of 0.85 or increasing them by a factor of 1.15, and then recalculating the derived parameters of the model accordingly (see Table 1 and Appendix B). The elasticities for group 3 remain at the standard values. The cost experiments involve decreasing the λ and φ parameters of the cost functions for TOU periods 1 and 2 (the periods with the highest marginal costs) by a factor of 0.85 or increasing them by a factor of 1.15. The cost parameters for TOU period 3 (the off-peak period) remain at the standard values.

Consider first the effects of altering the elasticities over the range from lowest to highest. In general, the effects of higher elasticities under a shift from fixed to shared TOU pricing exceed those under a shared-to-differentiated TOU transition. Peak to off-peak price ratios increase slightly under price differentiation when the elasticities are increased for groups 1 and 2: the ratio changes from 1.3 to 1.4 for group 1, from 1.8 to 1.9 for group 2. A similar result occurs under shared TOU pricing: the ratio there increases from 1.4 to 1.6. Whatever the shift of pricing regimes, and considering only the direction of effects, higher elasticities mean higher overall gains in welfare, consumption, and output as a result of a shift, and greater reductions of average cost and the proportion of output produced in the peak TOU period. For individual household groups the effects are mixed; again, the largest effects of higher elasticities occur when flat rates are replaced by shared TOU rates.

Now consider the effects of changes in cost parameters, again over the range from lowest to highest. At least for the ranges considered here, the effects of higher marginal costs on welfare, consumption, and output are larger than the effects of higher elasticities, most notably for welfare. For example, in moving from flat to differentiated prices, under higher marginal costs, the overall welfare increase is 6.5 percent, rather than 5.3 percent under standard cost parameters. Replacing a flat pricing scheme with a shared TOU scheme mitigates the effect of the higher marginal costs on welfare. Welfare thus increases more than it would otherwise as a result of the pricing shift. It would increase even more by a subsequent shift to a differentiated scheme, which would allow households additional flexibility in responding to the higher marginal costs. The same pattern holds for consumption and output, though in lesser degree. The peak to off-peak price ratios are higher for all groups in response to the changes in cost parameters when differential pricing replaces flat pricing: from 1.3 with low cost parameters to 1.5 with high cost parameters for group 1, 1.8 to 2.0 for group 2, and 1.4 to 1.8 for group 3. (The corresponding change in the peak/off-peak price ratio when shared TOU pricing replaces flat pricing is from 1.4 with low cost parameters to 1.8 with the high parameters.) A consistent pattern is observed on the supply side: higher marginal costs result in greater savings in terms of average cost when one pricing scheme is replaced by another, and a further reduction in the share of peak period production.

X. A TARGETED STRATEGY FOR THE HOUSEHOLDS WITH LOWEST DISCRETIONARY EXPENDITURE

We consider next a possible modification of the model that would target households with the lowest discretionary expenditure on electricity (group 1). (We interpret these as low income households but households could have low discretionary expenditure for other reasons – living in unusually hot or cold areas requiring more committed expenditure on heating or cooling, for example.) The aim of the modification is to ensure for such households a sufficiently large gain in welfare resulting from a change in pricing scheme. In the absence of special treatment, group 1 households would have the smallest electricity welfare increase in going from flat to shared TOU pricing (Table 2) or flat to differentiated TOU pricing (Table 4), and they would actually lose electricity welfare in going from shared to differentiated TOU pricing (Table 3). The following strategy is presented simply as an example of how the model could be modified so as to offset the relatively disadvantageous effects on one group by the dominance of others (especially group 3) in the determination of a market equilibrium. This is in keeping with regulatory bodies giving special consideration to lower income groups in rate setting. "Lifeline rates" are an example of that (see Berg and Roth, 1976, Berg and Herden, 1976, Shihag et al., 2004, Prasad, 2008, Price and Pham, 2009); another is the rebates for low income consumers under the Ontario Electricity Support Program (Ontario Energy Board, 2016) introduced by the Ontario Energy Board to offset increases in fixed distribution costs, starting in January 2016.

Assume then that group 1 has been identified (by a regulatory agency) as a lower income group deserving of separate treatment. (In practice, the lower income group might be determined based on taxable income criteria, as with the IRS tax return transcripts specified in the guidelines of the CARE Program of Pacific Gas and Electric (Pacific Gas &Electric, 2016), or the Lifeline Rate Program of the City of Olympia, Washington State (City of Olympia, 2016).) The incentive compatibility restrictions play a major role in restricting the range of feasible equilibrium solutions, with particular implications for those households with the least discretion in their choice of a TOU consumption pattern. If we can target an income group (in this case the lowest income group) with a specific price structure not available to other groups, we no longer require incentive compatibility restrictions for high income groups with respect to price structures offered to the lowest group. Therefore, some of the restrictions are removed, in particular the restrictions that require groups 2 and 3 to prefer their electricity prices to those of group 1. Groups 2 and 3 would know that special treatment of group 1 was in force and they would be required to accept prices for their groups that they might otherwise deem unacceptable (by comparison with those of group 1), and thus allow more price setting freedom to group 1. To put it differently, groups 2 and 3 would not be permitted to choose the price set offered to group 1. Group 1 would still be permitted comparisons with groups 2 and 3, but not the other way around. The original set of six incentive compatibility restrictions would now be reduced to four: two two-way comparisons between groups 2 and 3 and two one-way comparisons of group 1 with groups 2 and 3. Adapting the incentive compatibility inequalities in section II we have

$$v_k(x_k, p_k) \geq v_k(x_k, p_s) \qquad (k = 1,\ s = 2,3) \text{ and } (k, s = 2,3;\ s \neq k)$$

The results of adopting this strategy are shown in Table 5 when a shared TOU scheme is replaced by a differentiated scheme and a flat scheme is replaced directly by a differentiated scheme. As can be seen, welfare and consumption now increase for household group 1 at each transition; indeed the increases for group 1, the targeted group, now exceed those for either of the other two. The targeting and selective relaxation of incentive compatibility restrictions have had the intended effect. Moreover, this elimination of a subset of the incentive compatibility restrictions causes beneficial overall

Table 5. Moving from Flat Pricing and Shared TOU Pricing to Differentiated TOU Pricing Targeted for Lowest Discretionary Electricity Expenditure Households: The Effects on Prices, Welfare, Costs, Consumption, and Output

Prices and Costs

	Reference Flat	To Differentiated from Flat	Reference Shared TOU	To Differentiated from Shared TOU
Household 1				
p_1	1.000	1.372	1.189	1.485
p_2	1.000	0.945	1.131	1.023
p_3	1.000	0.630	0.828	0.672
$EVE/\overline{ME}$ (%)	0.0	10.4	0.0	3.8
Household 2				
p_1	1.000	1.211	1.189	1.198
p_2	1.000	1.323	1.131	1.297
p_3	1.000	0.768	0.828	0.759
$EVE/\overline{ME}$ (%)	0.0	4.7	0.0	1.8
Household 3				
p_1	1.000	1.559	1.189	1.535
p_2	1.000	1.070	1.131	1.054
p_3	1.000	0.727	0.828	0.719
$EVE/\overline{ME}$ (%)	0.0	5.9	0.0	2.7
Overall $EVE/\overline{ME}$ (%)	0.0	6.5	0.0	2.7
Production and Costs				
%ΔQ	-	2.32	-	1.21
%ΔC	-	-0.51	-	0.15
%Δ(C/Q)	-	-2.77	-	-1.05
C_1/C (%)	43.5	39.8	41.6	39.8
C_2/C (%)	25.3	24.7	24.8	24.7
C_3/C (%)	31.2	35.5	33.7	35.5

Consumption and Output

	Reference Flat	To Differentiated from Flat	Reference Shared TOU	To Differentiated from Shared TOU
Household 1				
$\%\Delta q_1$	-	-9.59	-	-5.85
$\%\Delta q_2$	-	-0.08	-	1.68
$\%\Delta q_3$	-	16.68	-	7.68
$\%\Delta q$	-	5.06	-	2.59
Household 2				
$\%\Delta q_1$	-	-4.18	-	-0.38
$\%\Delta q_2$	-	-4.49	-	-2.05
$\%\Delta q_3$	-	7.61	-	2.52
$\%\Delta q$	-	1.84	-	0.82
Household 3				
$\%\Delta q_1$	-	-5.76	-	-3.00
$\%\Delta q_2$	-	-1.18	-	0.43
$\%\Delta q_3$	-	6.43	-	2.96
$\%\Delta q$	-	1.81	-	1.04
Overall TOU Shares				
Q_1/Q (%)	25.3	23.2	24.2	23.2
Q_2/Q (%)	22.6	21.7	22.0	21.7
Q_3/Q (%)	52.1	55.1	53.9	55.1

Note: All parameters of the model have standard values if not otherwise indicated (see Table 1); p_1, p_2, p_3 are electricity rates for peak, mid-peak, and off-peak TOU periods, in that order; households 1,2,3 are lowest, medium, and highest discretionary electricity expenditure households, in that order. See text for definitions of welfare indicator $(EVE/\overline{ME})$ and symbols related to output (Q), cost (C), and consumption (q).

welfare gains (e.g., a gain in electricity welfare of 6.5 percent versus 5.3 percent in moving from flat to differentiated prices).

Another point to note is that here the path of transition matters. Going directly from flat rates to differentiated TOU rates yields a total electricity welfare gain of 6.5 percent. However, going from flat rates first to shared TOU rates, and then on to differentiated rates, would yield an electricity welfare gain of 6.0 percent (combining the results of Table 2a and 5). As well, the distributional impacts across customer groups are very different, depending on the path. Group 1 shows a gain in electricity welfare of 10.4 percent in going directly from flat to differentiated rates. On the other hand, the gain for that group is 5.5 percent in going from flat rates to shared TOU rates, and then on to differentiated rates. The fact that the differentiated price menus are different, depending on the initial price regime, provides evidence that the path matters, at least with this particular strategy. A policy implication is that for a jurisdiction starting with flat rates, but an interest in using such a strategy, it would be better to go directly to differentiated rates rather than taking a path involving a first move to shared TOU rates.

XI. SUMMARY OF THE SIMULATION RESULTS

We have explored and compared the effects of three alternative electricity pricing schemes: flat rates, shared TOU rates, and differentiated TOU rates. Proportional consumption changes resulting from a shift from one scheme to another are generally of small order but nevertheless can have substantial effects on prices, welfare, and production costs. Moving from flat to shared TOU prices increases welfare and consumption overall and for all household groups. Moving then from shared to differentiated TOU rates results in further overall increases but there are uneven distributional effects; under the basic specifications of the model only households with greater discretionary expenditure share in the increases. However, these effects can be offset by modifying the model to provide special treatment for low discretionary households. This special treatment, whereby the lowest discretionary expenditure group has access to a pricing option that other groups do not, leads to a substantial increase in welfare for the target group along with an increase in overall welfare. Flat to shared and shared to differentiated TOU pricing transitions both result in greater output, lower average costs, and a shift away from high cost peak TOU production. Assuming higher demand price elasticities or higher marginal cost parameters generally increases the effects of a transition from one pricing scheme to another.

XII. CONCLUDING REMARKS

Experiments with our theoretical general equilibrium model suggest strongly that there are possibilities for increasing welfare in a residential electricity market by incorporating consumer choice into the setting of TOU rates. The model itself is sufficiently flexible to allow modification and alternative calibration: utility and welfare functions could be specified differently; fixed costs on the supply side could be introduced and nonvolumetric charges on the consumer side; demand functions could be specified differently; elasticities and committed consumption levels could be calibrated in other ways; and so on. We would certainly not argue that the details of our model have exclusive rights for consideration. But we think that the central idea has much merit – the allowance for consumer choice of TOU rates, that is, subject to restrictions on both sides of the market necessary to ensure its viability. Our aim has been to demonstrate that merit and to that end we have made comparisons with other rate setting schemes and explored the sensitivity of the model results to reasonable alternative calibrations of demand and cost parameters. (We think too that the consumer choice idea may have applicability in other markets in which time of use is a consideration and pricing to smooth out the time pattern of load distribution a desirable goal.)

We have noted that the vast majority of electricity suppliers in North America still operate with flat residential rates. (Ontario is a notable exception.) Our results suggest that while there may be potential welfare gains in moving to shared TOU rates there could be greater gains in moving further, to some form of differentiated rates, and gains too for a supplier already on shared TOU rates. A number of obvious practical issues would have to be considered in that regard, of course. The model assumes that a supplier (or regulatory board to which it is responsible) has perfect knowledge of the preference patterns of different groups of consumers. "Perfect knowledge" would have to be replaced by "best estimates", based on customer surveys and econometric estimation of demand elasticities for heterogeneous household groups. (*Conjoint Analysis*, as developed originally in mathematical psychology, and applied in marketing studies and operations research, provides useful procedures for surveying and evaluating consumer preferences in situations similar to this one; see for example Louviere, Hensher, and Swait [2000], Hensher, Rose, and Greene [2005], Orme [2006].) There is also the issue of how well customers themselves can predict, in a survey, what their own preferences would be when initial choices are replaced by actual consumption realizations (see, for example, Miravete, 1996); one can imagine that a first round of chosen differentiated TOU rates might be replaced after some time (say a year) by a second round in which consumers could revise their choices, based on their first round experience. In short, the transition from theory to practice for differentiated TOU rates presents challenges but the potential benefits are there to be realized.

APPENDIX A: PROOF OF SEPARABILITY

Consider $v_k = x_k/P_k = \frac{y_k - p_k'\gamma_k}{P_k}$.

Using Roy's identity:

$$q_{kj} = -\frac{\frac{\partial v_k}{\partial p_{kj}}}{\frac{\partial v_k}{\partial y_k}}$$

$$= \frac{\frac{\gamma_{kj}}{P_k} + \frac{x_k \frac{1}{\theta_k}(\alpha_k(\bar{p}_k)^{\theta_k} + (1-\alpha_k)(p_0)^{\theta_k})^{\frac{1}{\theta_k}-1}\alpha_k\theta_k(\bar{p}_k)^{\theta_k-1}\frac{\beta_{kj}\prod_{i=1}^{3}(p_{ki})^{\beta_{ki}}}{p_{kj}}}{(P_k)^2}}{\frac{1}{P_k}}$$

This implies (with $P_k = (\alpha_k(\bar{p}_k)^{\theta_k} + (1-\alpha_k)(p_0)^{\theta_k})^{1/\theta_k}$ and $\bar{p}_k = \prod_{j=1}^{3}(p_{kj})^{\beta_{kj}}$)

$$q_{kj} - \gamma_{kj} = \frac{\beta_{kj}\alpha_k x_k\left(\frac{\bar{p}_k}{P_k}\right)^{\theta_k}}{p_{kj}} \qquad \text{(A.1)}$$

Now, with $\beta_{k1} + \beta_{k2} + \beta_{k3} = 1$, define

$$xe_k = \sum_{j=1}^{3} p_{kj}\left(q_{kj} - \gamma_{kj}\right)$$

$$= \sum_{i=1}^{3} \beta_{ki}\alpha_k x_k \left(\frac{\bar{p}_k}{P_k}\right)^{\theta_k}$$

$$=\alpha_k x_k \left(\frac{\bar{p}_k}{P_k}\right)^{\theta_k}$$

Therefore, using (A.1)

$$q_{kj} - \gamma_{kj} = \frac{\beta_{kj}\, xe_k}{p_{kj}}\,.$$

Notice that $\frac{\partial ln(q_{kj}-\gamma_{kj})}{\partial lnp_0} = \frac{\partial lnxe_k}{\partial lnp_0}$ for all $j = 1,2,3$.

This in turn implies that there exists $ve_k = \frac{xe_k}{\bar{p}_k}$, meaning that

$$q_{kj} - \gamma_{kj} = -\frac{\frac{\partial ve_k}{\partial p_{kj}}}{\frac{\partial ve_k}{\partial xe_k}} = \frac{\frac{xe_k}{(\bar{p}_k)^2}\left(\frac{\beta_{kj}\prod_{i=1}^{3}(p_{ki})^{\beta_{ki}}}{p_{kj}}\right)}{\frac{1}{\bar{p}_k}} = \frac{\beta_{kj}\, xe_k}{p_{kj}}$$

APPENDIX B: ADDITIONAL NOTES ON CALIBRATION

The calibration of all parameters is done at reference quantities corresponding to flat rates. Thus, for the following calibration derivations, the quantities in this appendix correspond to reference quantities. Calibration and optimization were implemented using Maple, version 17 with Maple Global Optimization Toolbox.

A. Demand Calibration

Determination of $\boldsymbol{\beta_{k1}}, \boldsymbol{\beta_{k2}}$, and $\boldsymbol{\beta_{k3}}$:

Holding electricity expenditure constant,

$$\frac{dln(q_{ki}-\gamma_{ki})}{dlnp_{ki}} = \left(\frac{q_{ki}}{q_{ki}-\gamma_{ik}}\right)\cdot\frac{dlnq_{ki}}{dlnp_{ki}}. \qquad \text{(B.1)}$$

Consistent with the notation of the text, define, $\eta_{ki} = \frac{dlnq_{ki}}{dlnp_{ki}}$.

In the context of our LES electricity system,

$$\frac{dln(q_{ki}-\gamma_{ki})}{dlnp_{ki}} = -1 + \beta_{ki}. \qquad \text{(B.2)}$$

Therefore, using equations (B.1) and (B.2)

$$\left(\frac{q_{ki}}{q_{ki}-\gamma_{ki}}\right) = \frac{-1+\beta_{ki}}{\eta_{ki}}$$

or $\gamma_{ki} = a_{ki} \cdot q_{ki}$ with

$$a_{ki} = 1 - \left(\frac{-1+\beta_{ki}}{\eta_{ki}}\right)^{-1}. \qquad \text{(B.3)}$$

As well, evaluating expenditures at reference flat prices, $\frac{q_{ki}-\gamma_{ki}}{q_{kj}-\gamma_{kj}} = \frac{\beta_{ki}}{\beta_{kj}}$ and $\frac{q_{ki}(1-a_{ki})}{q_{kj}(1-a_{kj})} = \frac{\beta_{ki}}{\beta_{kj}}$ which means

$$\frac{q_{ki}}{q_{kj}} = \frac{\beta_{ki}}{\beta_{kj}} \cdot \frac{(1-a_{kj})}{(1-a_{ki})} = \frac{\beta_{ki}}{\beta_{kj}} \cdot \frac{\left(\frac{\eta_{kj}}{-1+\beta_{kj}}\right)}{\left(\frac{\eta_{ki}}{-1+\beta_{ki}}\right)}. \qquad \text{(B.4)}$$

At reference prices, define $\frac{q_{k3}}{q_{k1}} = R31_k$ and $\frac{q_{k2}}{q_{k1}} = R21_k$.

It follows that with equation (B.4) and the adding-up condition for the βs, the following system of equations must hold:

$$(-1+\beta_{k1}) \cdot \beta_{k1} \cdot R21_k - \left(\frac{\eta_{k1}}{\eta_{k2}}\right) \cdot \beta_{k2} \cdot (-1+\beta_{k2}) = 0$$

$$(-1+\beta_{k1}) \cdot \beta_{k1} \cdot R31_k - \left(\frac{\eta_{k1}}{\eta_{k3}}\right) \cdot \beta_{k3} \cdot (-1+\beta_{k3}) = 0 \qquad \text{(B.5)}$$

$$\beta_{k1} + \beta_{k2} + \beta_{k3} = 1$$

Equation set (B.5), a system of three equations is solved for β_{k1}, β_{k2}, and β_{k3}.

Determination of $\boldsymbol{\gamma_{k1}}, \boldsymbol{\gamma_{k1}}$, and $\boldsymbol{\gamma_{k1}}$,:

As noted in equation (B.3),

$a_{kj} = 1 - \left(\frac{-1+\beta_{kj}}{\eta_{kj}}\right)^{-1}$ for $j = 1,2,3$ and $k = 1,3$.

Now, with quantities evaluated at reference prices $q_k = \sum_{j=1}^{3} q_{kj}$, it follows that

$q_{k1}\left(1 + \frac{q_{k2}}{q_{k1}} + \frac{q_{k3}}{q_{k1}}\right) = q_k$. q_k is an input.

Using the above $q_{k1} = q_k \cdot (1 + R21_k + R31_k)^{-1}$ and $q_{kj} = Rj1_k \cdot q_{k1}$ for *j=2,3.*

This allows us to calculate

$$\gamma_{ki} = a_{ki} \cdot q_{kj} \tag{B6}$$

Determination of α_k:

Now, at reference prices, $\alpha_k = \frac{q_k - \sum_{j=1}^{3} \gamma_{kj}}{y_k - \gamma_{k0} - \sum_{j=1}^{3} \gamma_{kj}}$ (B7)

Determination of θ_k:

Now, $\frac{\partial ln\left(q_k - \sum_{j=1}^{3} \gamma_{kj}\right)}{\partial lnP_k} = \left(\frac{q_k}{q_k - \sum_{j=1}^{3} \gamma_{kj}}\right) \cdot \eta_k$ where $\eta_k = \frac{\partial lnq_k}{\partial lnP_k}$

Also, based on the CES relationship between aggregate electricity and other goods, $\frac{\partial ln\left(q_k - \sum_{j=1}^{3} \gamma_{kj}\right)}{\partial lnP_k} = \theta_k(1 - \alpha_k) - 1.$

Thus, $\theta_k = \frac{\left(\frac{q_k}{q_k - \sum_{j=1}^{3} \gamma_{kj}}\right) \cdot \eta_k + 1}{1 - \alpha_k}$ (B8)

In summary, the inputs into the Demand Calibration are

y_k , $\gamma_{k0}, q_k, \eta_k, \eta_{kj}, R21_k,$ and $R31_k$ for $j = 1,2,3$ and $k = 1,2,3$, where q_k is the reference consumption at reference prices.

B. Supply Calibration

Determination of λ_j and ϕ_j :

We start by assuming that the hourly marginal cost for time period *j* at the quantity $\frac{\sum_{k=1}^{3} \gamma_{kj}}{52 \cdot HR_j}$ (corresponding to minimum committed consumption) is $\underline{mc}_j$ and the hourly marginal cost for time period *j* at the quantity $4 \cdot \left(\frac{\sum_{k=1}^{3} \gamma_{kj}}{52 \cdot HR_j}\right)$ is $\overline{mc}_j$. The slope of the marginal cost schedule between these two points is

$$\phi_j^* = \frac{\overline{mc}_j - \underline{mc}_j}{3 \cdot \left(\frac{\sum_{k=1}^{3} \gamma_{kj}}{52 \cdot HR_j}\right)}.$$

It follows that the y intercept is

$$\lambda_j^* = \underline{mc}_j - \phi_j^* \cdot \left(\frac{\sum_{k=1}^{3} \gamma_{kj}}{52 \cdot HR_j}\right).$$

Finally, the λ_j and ϕ_j are normalized such that the revenue and costs are equal at reference prices. That is $\lambda_j = \tau \cdot \lambda_j^*$ and $\phi_j = \tau \cdot \phi_j^*$ where

$$\tau = \frac{\left(\sum_{k=1}^{3} q_k^r\right)}{\sum_{j=1}^{3} cc_j}$$

with

$$cc_j = 52 \cdot HR_j * \left[\lambda_j^* * \left(\frac{\sum_{k=1}^{3} \gamma_{kj}}{52 \cdot HR_j}\right) + \phi_j^* * \left(\frac{\sum_{k=1}^{3} \gamma_{kj}}{52 \cdot HR_j}\right)^2\right]$$

Aside from the inputs coming from the Demand Calibration, to implement the above supply calibration we require the inputs $\underline{mc}_j, \overline{mc}_j$ and HR_j for *j=1,2,3.*

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