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QSEP Research Report No. 460

McMaster University
SOCIAL SCIENCES

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December 2016

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ABSTRACT

Immigrants can mix with the population of a receiving country in various ways. We consider *demographic* mixing by which we mean cross-mating, and more particularly the bearing of children where one parent is of immigrant descent and the other is not – *cross-parenting* as we term it. We consider a hypothetical country with an initial stable population and introduce immigration. The results of cross-parenting are taken into account by identifying three separate populations within the overall total: non-immigrant population, immigrant population (immigrants and their descendants), and mixed population. We develop a stylized model to track the three populations, with interest focusing in particular on how the proportion of mixed population changes through time as it moves toward a steady state. The model has a stable projection (Leslie) matrix that holds for all three populations and moves them forward from generation to generation as each evolves in its own way. As cross-parenting occurs the resulting progeny are transferred from the other populations to the mixed population. The pattern of cross-parenting is determined in the first instance by a matrix representing preferences among the three populations and alternative preferential patterns are experimented with, ranging from complete isolation to indifference as to cross-parenting choices. However the matrix must be modified to recognize supply constraints imposed by the sizes of the available populations and a restricted least-squares procedure is employed to effect the modification while remaining as close as possible to the original preference pattern. Alternative rates of immigration are experimented with also.

JEL: F22

Keywords: immigration, population mixing, cross-parenting, demographic modeling, parenting preferences

October 2016

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1. INTRODUCTION

There are many ways in which the two populations can mix when a home country receives immigrants. A rough (and perhaps overlapping) categorization might be as follows: *economic mixing* (working or doing business with each other); *social mixing* (belonging to the same common-interest groups, including religious institutions, schools, clubs, and informal friendship affiliations; *geographic mixing* (through neighbourhood proximity and contacts); and *demographic mixing* (mating and the bearing of children). Our concern in this paper is demographic mixing, and more specifically the bearing of children with mothers and fathers from different populations. Population mixing for us will be the result of *cross-parenting*, as we shall call it. (We use cross-parenting in preference to cross-breeding, the term common in biology, as it has a greater connotation of voluntary choice. Note too that the term population mixing in our context differs from its definition in epidemiology where it refers to contacts among people as a result of spatial movement; see Law et al., 2008.)

In the analysis that follows we will define three populations, a non-immigrant population, an immigrant population (original immigrants and their descendants), and a mixed population, and we will employ a simplified or "stylized" demographic projection model to trace the evolution of each population from one generation to the next under alternative assumptions about the propensity to cross-parent. Although implicit rather than explicit in the model immigrants could have the same characteristics as non-immigrants or they could differ in various ways – ethnicity, language, education, and others – and such differences could have an important bearing on the mixing pattern. While not directly related to immigration (except historically) we note that many studies of population mixing in the United States have focused on racial intermarriage. Fryer (2007), for example, contrasts the "extraordinary convergence" in relation to black-white economic and political empowerment with much less convergence "in the most intimate spheres of life – religion, residential location, marriage, and cohabitation" and observes that marriage across racial lines is a "rare event" (pp. 71, 72). Torch and Rich (2016) report that the proportion of black-white marriages and cohabitations among couples increased five-fold from 1980 to 2010 but still accounted for only 1.5 percent of the total (p. 1).

The integration of immigrants with non-immigrants is a concern for many countries (immigrants account for more than 20 percent of the population in some of the major recipients) and inter-mating, and cross-parenting are important indicators. Adserà and Ferrer (2015) provide a summary of the literature relating to intermarriage of immigrants and non-immigrants. They note that "the capacity to form and maintain exogamous unions (between native and foreign born) can be interpreted as the quintessence of successful integration" (p.324). Referring to the work of Duncan and Trejo (2007) they observe too that "selectivity into intermarriage influences ethnic identification", and hence the measurement of integration for those with "immigrant ancestry" (p. 324). Substituting "cross-parenting" for "intermarriage", these observations relate directly to our focus in the present paper.

The paper, based on a stylized model, is theoretical and exploratory. Our intention is to show how quickly, in generational time, one could expect integration to take place under alternative assumptions about the propensity for immigrants and non-immigrants to mix, where cross-parenting is the mixing instrument. We develop the model, calibrate it using life table data, and carry out a series of experiments starting with a stable population and introducing immigration at different rates. Alternative underlying mixing preferences are assumed in the experiments and a procedure is developed for adapting the preferences to render them consistent with restrictions imposed by the available populations of childbearing age. The preferences range from no mixing at all (no cross-parenting) to free, unrestricted mixing. The non-immigrant, immigrant, and mixed components of the overall population are projected as a tri-population system. Of special interest is the proportion of mixed in the total as it changes from one generation to the next and approaches a steady-state limit as the number of generations increases without bound.

2. A FRAMEWORK FOR THE MODEL

We start with a hypothetical country (region) with overall population N. The population is distributed among two sexes and five age groups: "children" (0 – 19), "young adults" (20 – 39), "middle aged" (40 – 59), "old" (60 – 79), and "very old" (80 – 99), with no survivors at 100. The age-sex distribution is captured by a 10-element vector n = $[n_x]$, with x = 1 to 5 representing the female age groups, x = 6 to 10 the male age groups. All children have young adult parents – mothers in group n_2, fathers in n_7. The fertility rate for young adult females is thus equivalent to the conventional total fertility rate. In addition, restricting fathers to the same age range allows the calculation of a male fertility rate (see below).

In the absence of migration, in or out, the population is augmented by births, depleted by deaths, has a 10 x 10 projection (Leslie) matrix Q, and a stable age distribution. The female fertility rate is set at the natural replacement level, survival rates are unchanging, and $n_{t+1} = Qn_t = n_t$ for all t where the time interval is 20 years, the same as the age intervals. For convenience we shall refer to this interval as a *generation.* (To keep notation as simple as possible we add a time subscript when necessary but avoid doing so otherwise.)

Now introduce immigration (still with no emigration, which we shall take to be zero) and assume that immigrants have the same stable age-sex distribution, proportionately, as the original non-immigrants and the same projection matrix Q. In our simulations below immigration may be one-time or repeated but to develop the framework assume for the present that it is a one-time event with immigrants arriving at t = 0. The question of interest is how rapidly will the populations of immigrants and non-immigrants mix where mixing in our context means cross-parenting – initially the bearing of children with one immigrant parent and one non-immigrant parent, although the descendants of such a union will also be regarded as mixed. (It is perhaps well to recognize, before proceeding, that realistically in virtually any place in the world all of the population will be descended from immigrants if one goes back far enough in time. In what follows we will simply identify t = 0 as a point at which new entrants will be classified as immigrants and the existing population as non-immigrants.)

We identify then three separate populations within the overall population N, each evolving in its own way: (1) the original non-immigrant population and its non-mixed descendants H; (2) the population of immigrants and their non-mixed descendants M; (3) a mixed population U, including all children of mixed lineage – children of mixed parents, grandchildren of mixed parents, and in general, all persons with lineage traceable back to a mixed union. (Mnemonically, H is for "home", M for "migrant", U for "union". Where the meaning is clear we shall sometimes use the word "immigrants" to refer to members of the M population, thus including both those who immigrated originally and their descendants.) These three populations have age-sex vectors h, m, and u corresponding in structure to n (and aggregating to n). They also have the same projection matrix Q. While initially immigrants and non-immigrants have the same proportionate age-sex distribution they may differ in other characteristics. The non-immigrant population will be augmented in each generation by births and depleted by deaths but those births to non-immigrant mothers mated with immigrant or mixed fathers will be transferred (reclassified) to the mixed population. Similarly, the immigrant population will be augmented by births and depleted by deaths but all births to immigrant mothers mated to non-immigrant or mixed fathers will be transferred to the mixed population. The mixed population will be augmented by births, depleted by deaths, and augmented also by the cross-parenting transfers from the other populations. If cross-parenting continues freely and indefinitely – if individuals choose to mate randomly and bear children without preference as to population membership – the non-immigrant and immigrant populations will vanish in the limit; all residents of the country will eventually be of mixed lineage. The proportion of mixed population in the total population of the country serves as an indicator at any given time of the degree of mixing that has occurred. (Note that the overall population N continues to have the same stable age-sex distribution; that is not affected by transfers among its component populations.)

The foregoing assumes one-time immigration. If immigration is repeated – at a constant rate proportional to the total population, let us say – the framework is the same as before except that the immigrant population will now be augmented by new immigrants each generation. The non-immigrant population will still vanish, in the limit, under random parenting, but the immigrant population will be continuously replenished and the mixed proportion in the overall population will always be less than one.

The accounting relations for the process with repeated immigration can be stated informally as follows. The change in the non-immigrant population from one generation to the next can be represented as

ΔH = Births – Deaths – CPT(H,M -> U) – CPT(H,U -> U)

CPT stands for a cross-parenting transfer of newborn children and the arrow indicates that the direction of transfer is to U, the mixed population. The transfers result from a non-immigrant/immigrant (mother/father) pairing (H,M) in the first case and a non-immigrant/mixed population pairing (H,U) in the second. (The first letter is always the population of the mother.) Using similar notation the changes in the immigrant and mixed populations can be represented as

ΔM = Births – Deaths – CPT(M,H -> U) – CPT(M,U -> U) + New Immigrants

ΔU = Births – Deaths + CPT(U <- H,M) + CPT(U <-H,U) + CPT(U <- M,H) + CPT(U <- M,U)

PREFERENTIAL DISTRIBUTION PATTERNS

Free and random mating/parenting yields one particular type of interpopulation distribution of children but there are others. We shall call these distributions *preferential* in the first instance. Preferential distributions reflect personal preferences and mutual mating/parenting agreements, and possibly also societal discrimination of one kind or another. Actual or *realized* distributions may differ from preferential ones by incorporating supply constraints imposed by the differing sizes of the populations and the limited availability of mating partners; preferential distribution patterns thus represent distributions as they would occur in the absence of such constraints. (Imagine for the moment a hypothetical situation in which there is a given number of young adult females but an unlimited number of males in each population so that any proportionate distribution of mothers according to the population of fathers is possible, and reflective only of preferences.) We shall deal with supply constraints shortly but first consider a 3 x 3 preferential distribution matrix $P = [p_{ij}]$. The rows of the matrix represent mothers in each of the three populations (H,M,U), the columns represent fathers from those populations. An element i, j represents the proportion of children who are born to mothers from population i and have fathers from population j, with the elements in each row summing to 1. This matrix can be configured in various ways to represent alternative preferential patterns. Here are some.

(1) *Indifference*: There is no preference in any of the three populations: the matrix is 3 x 3; p_{ij} = 1/3 for all i, j (this is what we called free and random parenting above).

(2) *Isolation*: There is no cross-parenting and hence no mixed population; non-immigrants parent with non-immigrants, immigrants parent with immigrants, and the matrix is reduced to 2 x 2: $p_{11} = p_{22} = 1$; $p_{12} = p_{21} = 0$. Isolation could be a matter of mutual preference or a consequence of societal discrimination (the two may be essentially equivalent).

(3) *Partial Discrimination*: Non-immigrants have a preference for parenting with non-immigrants but the preference is not exclusive - they will parent also with mixed population members, with lower probability, and with immigrants, with still lower probability; immigrants have a similar probability pattern, but in reverse; mixed population members are indifferent: $p_{11} > p_{13} > p_{12}$; $p_{22} > p_{23} > p_{21}$;

$p_{31} = p_{32} = p_{33} = 1/3$.

(4) *Adaptation*: The pattern is isolation at some initial time but moves toward indifference from one generation to the next - say linearly, for example - as society adapts and the integration of immigrants and non-immigrants comes to be fully accepted.

3. THE MALE FERTILITY RATE AND ASSOCIATED IMPLICATIONS

The requirement in the model that all parents must be young adults allows us to calculate a male fertility rate. The female fertility rate F is the ratio of live births to the number of young adult women, the same for all populations. Similarly, a male fertility rate G can be defined as the ratio of live births to young adult males, the number of live births being the same in both cases. With overall population vector n, the male fertility rate would be G = (n_2/n_7)F, where n_2 and n_7 are the numbers of young adult females and males, respectively. Assuming G to be the same for all populations (as is F), the overall number of births will be distributed according to the population of the fathers in proportion to the numbers of young adult males. Furthermore, the Q matrix incorporates a fixed sex ratio at birth and an adjustment to allow for infant and early childhood mortality in calculating the numbers of surviving male and female children in a population (see Appendix). The calculations are common to all three populations and so the distributions of male children and female children by population of father will be the same and proportional to the numbers of young adult males, as are births. They will be proportional also to the numbers of young adult females, for the same reasons, and thus any conflict between the two proportionality distributions must be resolved. Females from one population who would otherwise parent with males from another cannot do so if the latter population has an insufficient number of young adult males.

5. FROM PREFERENCES TO REALIZED PROPORTIONS

A preferential distribution provides a starting point but the relative availability of young adults in the three populations will dictate the feasibility of any cross-parenting pattern. To modify a preferential pattern to accommodate supply restrictions but remain as close as possible to the original preferential distribution we make use of a method known variously by the names *biproportional adjustment, iterative proportional fitting*, and others. The method was first given formal mathematical treatment by Deming and Stephan (1940), based on a restricted least squares criterion, in the context of the adjustment of sample estimates to fixed census marginal totals in the construction of contingency tables. It was subsequently taken up in the construction of economic input-output tables (Bacharach, 1965, 1970), where it has had extensive application over the years under the name *RAS Method*. (See Lahr and de Mesnard, 2004, for a history of RAS applications.) Note that under the isolation preferential pattern there are no supply constraints – the "demand" for and supply of young adult males are automatically in balance in each population so the procedure we are about to describe is required only for other patterns.

We wish to construct a 3 x 3 matrix B = $[b_{ij}]$, i, j = H,M,U, showing the realized distributions of children by mothers' population (rows) and fathers' population (columns). (B is 2 x 2 if there is no adult mixed population, as is the case temporarily when immigration is first introduced, and before any cross-parenting can occur.) Having constructed B we then want to convert it into a matrix A that is similar in structure to P but shows the actual (realized) proportionate distribution of births among fathers' populations for each population of mothers (row totals are thus equal to 1). The overall number of children, all populations combined, is given by calculation based on the Q matrix (all populations have the same Q matrix). The row totals of the B matrix ($b_{i\bullet}$) are calculated by distributing the overall number of children in proportion to the numbers of young adult females in the three populations and the column totals ($b_{\bullet j}$) by distributing the same overall number of children in proportion to the numbers of young adult males. Given some initial matrix B^* = $[b^*_{ij}]$, which in general will not satisfy the requirements that elements must sum to both row and column totals, the biproportional adjustment algorithm proceeds iteratively to find 3 x 3 (or 2 x 2) diagonal matrices D and C such that $B = D(B^*)C$ satisfies those requirements; it thus calculates B as a transformation of the initial matrix B^* by forcing row and column adding-up consistency. As a final step the A matrix is calculated by expressing each element of B as a proportion of its row total; P and A are thus comparable, P showing preferences, A showing realizations. The sex ratio of children is constant for each population, under the assumptions of the model, and thus the proportionate distributions provided by the A matrix are the same for male and female children.

The derivation of the B matrix is as follows. The row and column totals are calculated as above. The row totals are then distributed among the elements of each row in proportion to the corresponding elements of the P matrix, thus providing the initial matrix B^*, the starting point for the procedure. The elements of B^* sum to the correct row totals, by construction, but not (in general) to the column totals. They are then adjusted pro rata to force them to sum to the column totals but now they no longer sum to the row totals. They are forced again pro rata to sum to the row totals, and so it proceeds, iteratively, until convergence is obtained and the adding-up restrictions are satisfied. Following Deming and Stephan (1940) this simple procedure can be shown to be equivalent to minimizing the sum of squares of the differences between the B^* and B elements, $\sum(b^*_{ij} - b_{ij})^2$. (Deming and Stephan used a weighted average for illustration; we use an unweighted average, or a weighted average with weights equal to 1, if one prefers to say it that way.) The minimization is subject to the conditions that the rows of B must add to $b_{i\bullet}$ and the columns to $b_{\bullet j}$. The A matrix derived from B, and representing realized distributions, is thus as close as possible to the P matrix, representing preferential distributions, based on the least squares criterion. (The final adjustment factors that convert B^* to B in the iterative sequence are the diagonal elements of the D and C matrices – row adjustment factors for D, column adjustment factors for C.) The transformation of B^* to B (more fully, P to B^* to B to A) using the biproportional adjustment algorithm is unique and convergence is fast and guaranteed under simple conditions that are satisfied by the model.

The application of the biproportional adjustment algorithm here is similar to its application in the construction of an economic input-output matrix, as noted above: marginal input and output (row and column) totals are known precisely but interindustry product flows – the elements of the matrix – are not. They are represented initially by estimates that are then adjusted iteratively to enforce the adding-up restrictions. The present application involves only small matrices; input-output applications are generally on a much larger scale but the basic ideas are the same.

We provide below examples of the P matrices for selected preferential patterns and the corresponding derived A matrices for two of our immigration simulation applications. First though we lay out the formal specifications of the model on which the applications are based.

6. THE MODEL

The complete model can be derived as follows. In the absence of transfers of children the non-immigrant population would be projected one generation ahead as $h_{t+1} = Qh_t$. To introduce transfers we define a 10 x 10 matrix $R = [r_{ij}]$ where r_{12} and r_{62} are equal to q_{12} and q_{62} (the mortality-adjusted sex-specific fertility elements of Q – see Appendix) and all other elements are zero. RH_t is then a vector with the projected numbers of female and male children as the first and sixth elements, zeros elsewhere. Assuming A is a 3 x 3 matrix the proportion of children transferred out of the non-immigrant population is $a_{hm} + a_{hu} = 1 - a_{hh}$, the same for both sexes, the transfer coefficients being the elements of A representing particular cross-parenting proportions. In practice, the biproportional procedure is applied anew at each generation in a simulation, and thus A will vary. Time subscripts are therefore attached to the elements of A in the equations below in addition to the row and column subscripts. Putting all of this together, the full projection equation for the non-immigrant population can be written compactly in the form

$$h_{t+1} = (Q - (1 - a_{hht})R)h_t \qquad (1)$$

The projection for the immigrant population can be dealt with in the same way, with child transfer proportion $a_{mn} + a_{mu} = 1 - a_{mm}$, but now new immigrants must be added. Assume that new immigrants enter the country each generation as a fixed proportion φ of the population calculated as it would be *without* the new immigrants. They enter the country with the same age-sex distribution as the overall population, the vector of new immigrants is therefore $\varphi Q(h_t + m_t + u_t) = \varphi Q n_t$, and the projection equation for the immigrant population can be written as

$$m_{t+1} = (Q - (1 - a_{mmt})R)m_t + \varphi Q n_t \qquad (2)$$

(A small point: The immigration vector at generation t includes children who are immigrating with their parents. Alternatively, the children of immigrant parents could be born immediately *after* the parents enter the country - in the same generational interval, that is; that would make no difference. In either case the children would represent an addition to M, the immigrant population. φ would simply be defined to include both pre-entry and immediate post-entry births to immigrant parents.)

The projection equation for the mixed population, the recipient of child transfers, is then

$$u_{t+1} = Qu_t + (1 - a_{hht})Rh_t + (1 - a_{mmt})Rm_t \qquad (3)$$

Equations (1), (2), and (3) constitute the integrated tri-population system that we use to explore, by simulation, the rate of population mixing. Given an initial population h_{-1}, immigration commencing at t = 0 at an assumed rate φ, and a preferential cross-parenting pattern represented by P, A can be calculated, the system can be moved forward one generation at a time, and the proportionate distribution among the three populations noted. Of particular interest is the time path of the ratio of mixed to total population under alternative assumptions.

7. EXAMPLES OF P AND A MATRICES

For illustration, Tables 1 and 2 show examples of the P matrices for three preferential patterns, indifference, isolation, and a particular version of partial discrimination, along with the corresponding derived A matrices. One-time immigration is assumed in Table 1, repeated immigration in Table 2, with the immigration rate φ in both cases set at .20 (one of the immigration rates considered in simulations in the next section). The row elements of each matrix sum to 1 (rounding aside). For a P matrix each row represents the proportionate inter-population distribution among fathers of the total number of births to mothers in a given population, ignoring supply restrictions on the number of potential fathers (young adult males); For the A matrices a row represents the same thing but with the supply restrictions imposed. The P matrices remain the same from one generation to the next in the three examples shown in the table but the A matrices may change as the populations change. Under isolation the restrictions are automatically and continuously satisfied so A and P are always the same; for the other cases the table shows the A matrices after three generations (t = 3).

8. CALIBRATION OF THE MODEL

The model requires calibration for simulation application. The P matrix is chosen by assumption and the A matrix is then derived, as above. ϕ is also chosen by assumption. However Q must be specified realistically and for that purpose we use a projection matrix based on Canadian life tables centered on the year 2001 (Statistics Canada, 2006). Any realistic projection matrix would serve our purposes but this is one that we have used in previous studies (Denton and Spencer, 2014, 2015a, 2015b) and we find it convenient to use it here. The matrix is 10 x 10, representing females and males in the five broad age groups. It incorporates survival rates for those groups, a female fertility rate set at the natural replacement level consistent with the life tables (approximately 2.0745) and adjusted for infant and early childhood mortality, and a male/female sex ratio at birth of 1.05. The Q matrix and associated stable population age-sex distribution are provided in the Appendix, with discussion.

9. SIMULATIONS WITH ONE-TIME IMMIGRATION

We begin the simulations with ones that assume one-time immigration, as presented in Tables 3 and 4. The reported simulations span a period of eight generations (t = 0,1,…,7). The tables show the evolving percentage distribution of the population among its three components under alternative assumptions: alternative preferential patterns for a given immigration rate in Table 3; alternative immigration rates for a given preferential pattern in Table 4. (We refer to immigration *rate* even though it is applied only once here; *proportion* might be better but using the term rate allows a smoother transition in language from one-time to repeated immigration in the next section.) The immigration rates considered are 10, 20, and 30 percent (φ = .10, .20, and .30). These rates may appear large but they have to be interpreted relative to the generational time interval of the model, 20 years. Viewed on a per annum basis within that interval they are much smaller, of course, and probably not unrealistic in a more familiar context of popular discussions of immigration or immigration policy. Looking at them that way, the rates are approximately .48, .92, and 1.32 percent per annum. (These observations apply to one-time immigration here but more especially to the repeated immigration simulations in the next section.)

The immigration rate is set at φ = .20 in Table 3 and simulation results for four preferential patterns are presented: indifference, isolation, partial discrimination, and adaptation. The P matrices for indifference, isolation, and partial discrimination are as shown in Tables 1 and 2, and discussed above. The matrix for adaptation (also discussed above) assumes isolation at the beginning (t = 0) but moves linearly, from one generation to the next, until it achieves the indifference form after three generations (t = 3), which form it retains thereafter.

Under isolation there is no mixed population and no cross-parenting of immigrants and non-immigrants; the H/N and M/N proportions are unchanging over the eight generations (and beyond). Under the other preferential patterns the mixed population proportion is small at first in each case but steadily increasing as cross-parenting shifts the distribution and the H/N and M/N proportions decline; it is clear from Table 3 (and simple reasoning) that the ultimate percentage distribution as the number of generations increases without bound would be U/N = 100, H/N = M/N = 0 for all patterns except isolation. The most rapid shift in distribution occurs under indifference, as one would expect: the mixed population is almost a third of the total after three generations (U/N is 32.0 percent) and just under half after four generations (U/N is 49.5 percent). The slowest shift occurs under partial discrimination, as we have defined that pattern. One might think of indifference as providing a benchmark with which the results of other patterns can be compared.

The rate of immigration obviously has a major role in determining the generational pace of population mixing. Table 4 shows the shifts in population distribution for the indifference pattern under the three alternative immigration rates that we have chosen to experiment with. The differences in shift patterns are in general as one would expect: increasing the immigration rate from .20 to .30 produces more rapid shifting (U/N is 40.0 percent after three generations) and a faster approach to the limiting distribution; reducing the rate to .10 has the opposite and much slower effect (U/N is 19.6 percent after three generations).

10. SIMULATIONS WITH REPEATED IMMIGRATION

The second set of simulations assumes that immigration occurs at the rate φ in every generation. The results for these simulations are reported in Tables 5 and 6. Following the same plan as before, Table 5 shows results for alternative preferential patterns with a fixed immigration rate, Table 6 shows results for alternative immigration rates with a fixed preferential pattern.

One general feature of the simulations in Table 5 (with φ set at .20) is that the proportion of immigrants in the population increases over the eight generation span. The pace at which it increases tapers off though. M/N increases from the beginning but the increases get smaller from one generation to the next, and for the adaptation preferential pattern they are replaced by decreases, starting at t = 5, as the effects of indifference begin to offset the earlier effects of isolation in the adaptation process. The mixed proportion U/N rises in all cases except isolation, where of course it remains at zero.

Each preferential pattern results in an ultimate stable state for the proportionate population distribution as the number of generations increases without bound. The (proportionately) stable state for isolation is obviously M/N = 100 percent, H/N = 0. (The H population itself is unchanging; it simply becomes a smaller and smaller proportion of the total as the immigrant population grows.) We have calculated the stable states for the other preferential patterns (with $\varphi = .20$) by running the model for 20 generations (more than enough to achieve stability, for practical purposes). H/N is 0 in all cases. For indifference and adaptation M/N = 41.6 percent, U/N = 58.4; for partial discrimination M/N = 47.9 percent, U/N = 52.1. That the results for adaptation and indifference are the same is simply a consequence of the fact that once isolation is replaced by indifference in the adaptation process the effects of isolation wear off and eventually the evolution of the population follows the same course as it would under pure indifference.

Taking a shorter-run view, and focusing on the mixed proportion, U/N is 29.2 percent after three generations under indifference and 41.9 after four. As with one-time immigration, the percentages are smaller for the other patterns – 15.5 and 24.3 under partial discrimination, 19.6 and 32.9 under adaptation.

The rate of immigration plays an important role in determining the population distribution, as one would imagine, and as illustrated in Table 6 for the indifference pattern. However the actual results are somewhat different from what one might have expected. Lowering the rate reduces the mixed proportion somewhat in the earlier generations but increases it in the later ones, and especially in the final stable state. Setting φ to .10 rather than .20 reduces the U/N percentage from 29.2 to 23.9 after three generations but increases it from 58.0 to 77.1 in the stable state. Setting φ to .30 increases U/N in the first two generations but by the third it lowers it slightly, to 28.2 percent, and thereafter lowers it more sharply; in the stable state the percentage falls to 43.7. The reason for the differences between early effects and later ones is that it takes time for repeated immigration to build up the immigrant population, starting from a base of zero, and then time for the mixing process to take advantage of the presence of more immigrants. On the one hand, as the immigrant population continues to grow it forms an increasing share of the total population, how fast depending on the rate of new immigrant entrants. On the other hand, a larger immigrant population means more opportunities for mixed parenting, thus tending to raise the proportion of mixed population, and so reduce the proportion of immigrant population. These two offsetting effects eventually strike a balance and produce a stable state.

11. CONCLUDING REMARKS

We have used a much simplified demographic model to provide the results reported in this paper – a stylized model, as we have called it. There are many ways in which the model could be modified. The number of age groups could be increased and the time interval for a generation reduced; immigrants could be assigned fertility and mortality rates different from those of the non-immigrant population, and perhaps changing over time as the two populations mix; fertility rates could be set above or below the natural replacement level; heterogeneity of immigrants could be introduced and different populations created to accommodate immigrants of different types; different preferential parenting patterns could be specified and their effects explored. A reader can no doubt think of other possible modifications. These modifications would come at a cost of course in terms of complicating the model. Our aim in the present paper has been to provide a “big picture” view of how immigrant and non-immigrant populations might interact (or not interact) in the bearing of children of mixed parentage and how the overall population composition of a country might change accordingly, in generational time. With the “big picture” goal in mind we have kept the model as simple as possible.

REFERENCES

Adserà, Alícia and Ferrer, Ana (2015). Immigrants and demography: marriage, divorce, and fertility. Chapter 7 in *Handbook of the Economics of International Migration,* Volume 1A, Chiswick, Barry R. and Curtin, Paul W. (Editors), North Holland, pp. 315-374. ISSN 2212-0092, http://dx.doi.org/10.1016/B978-0-444-53764-5.00007-4

Bacharach, M. (1965). Estimating nonnegative matrices from marginal data. *International Economic Review* 6(3): 294-310.

Bacharach, M. (1970). *Biproportional Matrices and Input-Output Change*. Cambridge University Press.

Deming, W.E. and Stephan, F.F. (1940). On a least squares adjustment of a sampled frequency table when the expected marginal totals are known. *Annals of Mathematical Statistics* 11(4): 427-444. Stable URL: http://www.jstor.org/stable/2235722.

Denton, F.T. and Spencer, B.G. (2014). Exploring the population implications of male preference when sex probabilities at birth can be altered. *Demographic Research*. 31(25): 757-778. doi: 10.4054/DemRes.2014.31.25.

Denton, F.T. and Spencer, B.G. (2015a). Modelling the age dynamics of chronic health conditions: life-table-consistent transition probabilities and their application. *Canadian Journal on Aging* 34(2): 176-193. doi: 10.1017/S071498081500001X.

Denton, F. T. and Spencer, B.G. (2015b). A simulation analysis of the longer-term effects of immigration on per capita income in an aging population. *International Journal of Population Studies* 1(1): 75-93. http://dx.doi.org/10.18063/IJPS.2015.01.006.

Duncan, B. and Trejo, S. (2007). Ethnic identification, intermarriage and unmeasured progress by Mexican Americans. In: Jorjan, G. J. (Ed.), Mexican Immigration to the United States. National Bureau of Economic Research and the University of Chicago Press, Chicago, pp. 229-267.

Fryer, R. G. (2007). Guess who's been coming to dinner? Trends in interracial marriage over the 20th century. *Journal of Economic Perspectives* 21(2): 71-90. doi: 10.1257/jep.21.2.71

Hesketh, T. and Xing, Z.W. (2006). Abnormal sex ratios in human populations: causes and consequences. *Proceedings of the National Academy of Sciences*, 103: 13271-13275. http://dx.doi.org/10.1073/pnas.0602203103.

Lahr, M.L. and de Mesnard, L. (2004). Biproportional techniques in input-output analysis: updating and structural analysis. *Economic Systems Research*. 16(2): 115-134. doi: 10.1080/0953531042000219259.

Law, G.R., Feltbower, R.G., Taylor, J.C., Parslow, R.C., Gilthorpe, M.S., Boyle, P., and McKinney, P.A. (2008). What do epidemiologists mean by 'population mixing'? *Pediatric Blood Cancer,* 51: 155-160. doi: 10.1002/pbc.21570.

Statistics Canada (2006). *Life tables, Canada, provinces and territories 2000 to 2002.* Ottawa. Catalogue 84-537-XIE.

Torch, F. and Rich, P. (2016). Declining Racial Stratification in Marriage Choices? Trends in Black/White Status Exchange in the United States, 1980 to 2010. *Sociology of Race and Ethnicity*, May 28: 1-32, doi: 10.1177/2332649216648464

Table 1. Examples of P and A Matrices for Three Alternative Preference Patterns: One-Time Immigration							
	P matrix				A matrix		
	H	M	U		H	M	U
Indifference							
H	0.333	0.333	0.333		0.579	0.005	0.417
M	0.333	0.333	0.333		0.579	0.005	0.417
U	0.333	0.333	0.333		0.579	0.005	0.417
Isolation							
H	1.000	0.000	--		1.000	0.000	--
M	0.000	1.000	--		0.000	1.000	--
U	--	--	--		--	--	--
Partial disc.							
H	0.571	0.143	0.286		0.762	0.033	0.205
M	0.143	0.571	0.286		0.361	0.252	0.387
U	0.333	0.333	0.333		0.584	0.102	0.314

Note: Row entries are for mothers' population, column entries for fathers' population. The A matrices are for one-time immigration calculated after three generations (t =3) with φ = .20.

Table 2. Examples of P and A Matrices for Three Alternative Preference Patterns: Repeated Immigration							
	P matrix				A matrix		
	H	M	U		H	M	U
Indifference							
H	0.333	0.333	0.333		0.335	0.215	0.450
M	0.333	0.333	0.333		0.335	0.215	0.450
U	0.333	0.333	0.333		0.335	0.215	0.450
Isolation							
H	1.000	0.000	--		1.000	0.000	--
M	0.000	1.000	--		0.000	1.000	--
U	--	--	--		--	--	--
Partial disc.							
H	0.571	0.143	0.286		0.666	0.129	0.205
M	0.143	0.571	0.286		0.188	0.582	0.230
U	0.333	0.333	0.333		0.418	0.325	0.257
Note: Row entries are for mothers' population, column entries for fathers' population. The A matrices are for repeated immigration calculated after three generations (t =3) with φ = .20.							

Table 3. Component Populations as Percent of Total: Four Alternative Preference Patterns with One-Time Immigration (φ = .20)								
	Generation (t)							
	0	1	2	3	4	5	6	7
Indifference								
H/N	83.3	79.9	73.5	62.9	48.7	33.6	20.1	10.0
M/N	16.7	13.2	9.2	5.1	1.7	0.3	0.0	0.0
U/N	0.0	7.0	17.3	32.0	49.5	66.1	79.9	90.0
Isolation								
H/N	83.3	83.3	83.3	83.3	83.3	83.3	83.3	83.3
M/N	16.7	16.7	16.7	16.7	16.7	16.7	16.7	16.7
U/N	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Partial disc.								
H/N	83.3	81.6	78.1	71.9	63.0	52.0	39.9	27.7
M/N	16.7	15.0	12.4	8.9	5.3	2.6	1.1	0.3
U/N	0.0	3.4	9.5	19.1	31.7	45.3	59.1	72.0
Adaptation								
H/N	83.3	82.2	79.1	72.3	61.7	47.8	32.7	19.3
M/N	16.7	15.6	12.8	9.0	5.0	2.1	0.5	0.1
U/N	0.0	2.2	8.1	18.8	33.3	50.1	66.8	80.6
Note: All immigration takes place at t = 0. See text and Table 1 or 2 for specifications of P matrices for indifference, isolation, and partial discrimination. The matrix for adaptation changes linearly from isolation at t = 0 to indifference at t = 3 and remains fixed thereafter.								

Table 4. Component Populations as Percent of Total: Indifference Preference Pattern with One-Time Immigration (φ = .10, .20, .30)								
	Generation (t)							
	0	1	2	3	4	5	6	7
φ = .10								
H/N	90.9	88.8	84.9	77.8	67.3	53.9	38.8	24.5
M/N	9.1	7.0	4.8	2.6	0.8	0.1	0.0	0.0
U/N	0.0	4.1	10.3	19.6	32.0	46.1	61.2	75.5
φ = .20								
H/N	83.3	79.9	73.5	62.9	48.7	33.6	20.1	10.0
M/N	16.7	13.2	9.2	5.1	1.7	0.3	0.0	0.0
U/N	0.0	7.0	17.3	32.0	49.5	66.1	79.9	90.0
φ = .30								
H/N	76.9	72.5	64.7	52.4	37.3	23.1	12.1	5.0
M/N	23.1	18.6	13.2	7.6	2.8	0.6	0.1	0.0
U/N	0.0	8.9	22.1	40.0	59.9	76.3	87.8	95.0
Note: All immigration takes place at t = 0. See text and Table 1 or 2 for specification of P matrix for the indifference pattern.								

Table 5. Component Populations as Percent of Total: Alternative Preference Patterns with Repeated Immigration (φ = .20)								
	Generation (t)							
	0	1	2	3	4	5	6	7
Indifference								
H/N	83.3	66.5	49.4	32.6	18.3	8.6	3.2	0.8
M/N	16.7	27.7	34.6	38.3	39.8	40.6	41.0	41.2
U/N	0.0	5.8	16.0	29.2	41.9	50.9	55.9	58.0
Isolation								
H/N	83.3	69.4	57.9	48.2	40.2	33.5	27.9	23.3
M/N	16.7	30.6	42.1	51.8	59.8	66.5	72.1	76.7
U/N	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Partial disc.								
H/N	83.3	68.0	53.7	40.1	27.7	17.2	9.4	4.4
M/N	16.7	29.1	38.3	44.4	48.0	49.9	50.5	50.5
U/N	0.0	2.9	8.0	15.5	24.3	32.9	40.0	45.1
Adaptation								
H/N	83.3	68.5	53.9	38.1	23.5	12.2	5.0	1.5
M/N	16.7	29.6	38.3	42.3	43.6	43.4	42.7	42.2
U/N	0.0	1.9	7.7	19.6	32.9	44.4	52.3	56.2
Note: Immigration commences at t = 0. See text and Table 1 or 2 for specifications of P matrices for indifference, isolation, and partial discrimination. The matrix for adaptation changes linearly from isolation at t = 0 to indifference at t = 3 and remains fixed thereafter.								

Table 6. Component Populations as Percent of Total: Indifference Preference Pattern with Repeated Immigration (φ = .10, .20, .30)								
	Generation (t)							
	0	1	2	3	4	5	6	7
φ = .10								
H/N	90.9	80.8	68.8	54.3	38.3	23.5	12.1	5.0
M/N	9.1	15.5	19.6	21.8	22.5	22.7	22.8	22.9
U/N	0.0	3.8	11.6	23.9	39.2	53.8	65.0	72.1
φ = .20								
H/N	83.3	66.5	49.4	32.6	18.3	8.6	3.2	0.8
M/N	16.7	27.7	34.6	38.3	39.8	40.6	41.0	41.2
U/N	0.0	5.8	16.0	29.2	41.9	50.9	55.9	58.0
φ = .30								
H/N	76.9	55.8	36.7	21.0	10.0	3.8	1.1	0.2
M/N	23.1	37.4	46.2	50.9	53.2	54.6	55.5	56.1
U/N	0.0	6.8	17.1	28.2	36.8	41.6	43.5	43.7
Note: Immigration commences at t = 0. See text and Table 1 or 2 for specification of P matrix for the indifference pattern.								

APPENDIX: THE Q MATRIX AND THE STABLE POPULATION DISTRIBUTION

The Q matrix is shown in Table A1. Age group survival rates, elements (2,1), (3,2), etc., are derived from Canadian life tables, as noted in the text; infant/early-childhood survival rates - S_{f0} for females, S_{m0} for males - are derived also from those tables. The (female) fertility rate F is the natural replacement rate consistent with the survival rates and the assumed male/female ratio at birth. The latter ratio is set at 1.05 and the female and male proportions, C_f and C_m, are calculated accordingly. (As noted by Hesketh and Xing, 2006, p. 13271, "In the absence of manipulation, the sex ratio at birth is remarkably consistent across human populations, with 105 – 107 male births for every 100 female births." Our choice of 1.05 is approximately the long-standing Canadian ratio.)

The application of Q to an initial stable population vector n_t maintains the stability in perpetuity: $n_{t+k} = Q^k n_t = n_t$ for all $k \geq 0$. To obtain F experimentally and the corresponding stable age-sex distribution we projected a starting population vector repeatedly with alternative values of F for 100 generations until numerically satisfactory stability was achieved. (The starting population vector was calculated by combining the male and female life table populations.) The resulting stable vector is shown in Table A2, in approximate percentage distribution form.

Table A1.The Calibrated Q Matrix for a Stable Population

	Col.1	Col.2	Col.3	Col.4	Col.5	Col.6	Col.7	Col.8	Col.9	Col.10
Row1	0	$S_{f0}C_fF$	0	0	0	0	0	0	0	0
Row2	0.9942	0	0	0	0	0	0	0	0	0
Row3	0	0.9769	0	0	0	0	0	0	0	0
Row4	0	0	0.8635	0	0	0	0	0	0	0
Row5	0	0	0	0.3798	0	0	0	0	0	0
Row6	0	$S_{m0}C_mF$	0	0	0	0	0	0	0	0
Row7	0	0	0	0	0	0.9875	0	0	0	0
Row8	0	0	0	0	0	0	0.9617	0	0	0
Row9	0	0	0	0	0	0	0	0.7850	0	0
Row10	0	0	0	0	0	0	0	0	0.2575	0

Note: S_{f0} = 0.9940, S_{m0} = 0.9924; C_f = 0.4878, C_m = 0.5122; F = 2.0745

Table A2. The Initial Stable Population Vector (n): Elements as Percentages of Total				
Age group	Females		Males	
	Element	% of total population	Element	% of total population
Children	n_1	12.2	n_6	12.8
Young adults	n_2	12.1	n_7	12.6
Middle aged	n_3	11.9	n_8	12.2
Old	n_4	10.2	n_9	9.5
Very old	n_5	3.9	n_{10}	2.5