



**RESEARCH INSTITUTE FOR QUANTITATIVE  
STUDIES IN ECONOMICS AND POPULATION**

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ON MICRO AND AGGREGATE DATA:  
AN INTEGRATED FRAMEWORK AND EMPIRICAL EVALUATION**

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**QSEP Research Report No. 457**

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**March 2015**

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# **Biases in Consumer Elasticities Based on Micro and Aggregate Data: An Integrated Framework and Empirical Evaluation\***

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March 5, 2015

Policy analysis frequently requires estimates of aggregate (or mean) consumer elasticities. However, estimates are often made incorrectly, based on elasticity calculations at mean income. We provide in this paper an overall integrated analytical framework that encompasses these biases and others. We then use empirically derived parameter estimates to simulate and quantify the full range of biases. We do that for alternative income distributions and four different demand models. The biases can be quite large; they generally grow as the degree of income inequality rises, the underlying expenditure elasticity differs from one, and the rank of the model increases.

**Keywords:** Aggregate consumer elasticities, aggregation bias, consumer demand, income inequality, income distribution, model rank.

**JEL Classification:** D11, C43

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\* The helpful suggestions and comments of Arthur Lewbel and two referees are greatly appreciated.

# **Biases in Consumer Elasticities Based on Micro and Aggregate Data: An Integrated Framework and Empirical Evaluation**

## **1. INTRODUCTION**

Economic policy decisions frequently require the evaluation of aggregate responses - the aggregate response of consumer expenditure on gasoline to a gasoline tax, and the resulting tax revenue, for example, the aggregate response of expenditure on specific commodities to a change in the rate of income tax, or the aggregate effect of an income supplement on the demand for rental housing.<sup>1</sup> The empirical literature has many such examples (see Denton and Mountain, 2011a). Responses at the macro level are often represented best in the form of aggregate elasticities - or mean elasticities, since the two are the same. True mean elasticity formulas are seldom used, though. Elasticities are often reported at mean or other income levels for econometric models fitted to micro data, and elasticities calculated from aggregate data (and hence subject to aggregation bias) are often interpreted as elasticities at mean income. But elasticities at the mean are not the same as mean elasticities; they are approximations at best and fail to take into account the characteristics of the income distribution. In this paper, for models of arbitrary rank, and arbitrary income distribution, we derive formulas for the biases resulting from the use of elasticities at the mean (based on aggregate or micro data) to represent mean elasticities and from the use of aggregate rather than micro data in the calculation of either. We then calculate, by simulation, what the magnitudes of the biases might be in practice, using four alternative demand models. We calibrate the models using realistic parameter values taken from the empirical literature and calibrate the associated income distribution using parameters representing two alternative distributions with quite different degrees of inequality. We do this for both compensated and

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<sup>1</sup> In their survey on how to account for heterogeneity in aggregation, Blundell and Stoker (2005) begin their discussion by emphasizing that “some of the most important questions in economics...concern economic aggregates.” Economics “is often concerned with...aggregate consumption and savings, market demand and supply, total tax revenues,... and so forth.” Moreover, Slottje (2008) points to a recent “experiment of the US government in pumping over \$50 billion dollars into consumers’ hands to jump start the US economy in 2008” as an exemplification of “the importance of understanding aggregate consumer behavior and what does and does not impact it.”

uncompensated price elasticities, and for expenditure elasticities. We find from the simulation analysis that the biases can in fact be quite large.

There has been a long-standing concern in the literature about biases in the use of aggregate data to estimate micro demand parameters and elasticities (e.g., Gorman, 1953, Stoker, 1984, 1986, Blundell and Stoker, 2005). Numerical estimates of these biases for linear or quadratic ‘Almost Ideal Demand Systems’ have been provided by Blundell, Pashardes, and Weber (1993), Blundell, Meghir, and Weber (1993), and Denton and Mountain (2001, 2004). We incorporate biases of this kind into our framework and extend the list of models considered, but a novel focus is the derivation and quantification of the biases in the common practice of using elasticities at the mean (“representative consumer”) as if they were mean elasticities, whether based on micro data or aggregate data. We think that the exact definition and quantification of these biases is a useful contribution to the literature on consumer demand modeling and estimation, and one with practical relevance for consumer-related policy analysis. The paper provides a comprehensive framework encompassing all of the relevant biases and we think that too is a novel and useful contribution. Anticipating our simulation results, we find biases as high as 26 percent in elasticities incorrectly calculated – and quite aside, of course, from any errors in econometric estimation. In general, the biases increase with the rank of the model employed, the difference of the underlying expenditure elasticity from one, and the degree of inequality in the income distribution.

We begin, in the next section, with a taxonomy of practical situations involving estimation of elasticities with either micro or aggregate data. This provides the framework for the subsequent derivation and quantification of the elasticity biases.

## 2. A TAXONOMY OF ELASTICITY SITUATIONS AND BIASES

Three types of elasticities are of interest: expenditure elasticities, full (uncompensated) price elasticities, and compensated price elasticities.<sup>2</sup> With those

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<sup>2</sup> Full price elasticities may be of practical importance for policy forecasting - forecasting the revenue yield of a gasoline or cigarette tax, for example - while compensated price elasticities are of more interest from a welfare point of view.

elasticities in mind we consider the following classification of situations relevant for our purposes:

Situation 1: Micro data are available and are used correctly to calculate *mean* (or *aggregate*) elasticities.

Situation 2: Micro data are available and are used to calculate *elasticities at the mean of the income distribution*. Those elasticities at the mean are then used incorrectly as if they were *mean elasticities*.

Situation 3: Only aggregate data are available and those are used to estimate the underlying micro model and corresponding elasticities. The same elasticities are then used incorrectly as if they were *mean* (or “*representative consumer*”) *elasticities* in the micro model, and possibly to represent the aggregate effects of a price or income change.<sup>3</sup>

Figure 1 provides a schematic representation of these three situations and corresponding biases. We derive the formulas for calculating the mean elasticities in Situation 1 and the biases associated with calculating those elasticities in Situations 2 and 3. The biases depend on the income distribution, irrespective of whether micro data or aggregate data are used. There is an interesting exception though: calculations of mean expenditure elasticities based on aggregate data are unbiased; regardless of income distribution and model functional form, there is no aggregation error. Note that we follow the common practice in the literature of treating income and total expenditure as equivalent for purposes of analysis.

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<sup>3</sup>In spite of the increased availability and obvious advantages of micro data sets it is still the case that aggregate data must often be used in estimating consumer demand models. Of 21 published articles surveyed by the present authors, 15 used aggregate data in the estimation of “almost ideal demand systems,” either AIDS or QUAIDS (Denton and Mountain, 2011a). The reasons no doubt vary: lack of availability of micro data for a particular country or region, lack of sufficient commodity detail required for a particular purpose, or of observations on particular explanatory variables, the need to use time series available only at the aggregate level in order to estimate a model with dynamic properties, and so on. We note too that much of the attention given to elasticities calculated from aggregate data in the literature has focused on their use as estimates of underlying micro elasticities, rather than as estimates of mean or aggregate elasticities, even though the latter are often of greater policy relevance.

We label a mean elasticity calculated correctly, using micro data, as ME in Situation 1, as shown in Figure 1. (ME is the bias-free “gold standard” in our framework.) An elasticity at mean income calculated using micro data in Situation 2 is labeled as EM, and an elasticity at mean income calculated using aggregate data in Situation 3 as AM. The figure indicates, by arrows, the incorrect interpretations and corresponding presence of bias: the bias in using EM to represent ME, the bias in using AM to represent EM, and the bias in using AM to represent ME.

As we have noted, the traditional focus of the literature has been on the biases in using aggregate data to estimate micro parameters and elasticities (interpreting Situation 3 as if it were Situation 1). We add to that, in the present paper, a focus on the biases in estimating mean elasticities (correctly defined in Situation 1), whether they be from elasticities calculated at mean income with micro data (Situation 2) or from elasticities calculated at mean income from aggregate data (Situation 3). (To date, these biases have not been derived explicitly in the literature, or quantitatively explored.) Along with the traditional aggregation biases, this additional focus provides a more comprehensive framework for sorting out the various issues involved and quantifying the effects.

### 3. FRAMEWORK FOR THE ANALYSIS

The analysis assumes  $I$  commodities, indexed by  $i$ ,  $K$  households, indexed by  $k$ , and a common price vector  $p = (p_1, p_2, \dots, p_I)$  (a situation sometimes referred to as “the law of one price”). Household  $k$  spends  $x_{ik}$  units of income to purchase  $q_{ik} > 0$  units of commodity  $i$ , has total expenditure (equivalently, income)  $x_k$ , and thus an expenditure share  $w_{ik} = \frac{x_{ik}}{x_k}$ . Now consider the generic expenditure system

$$w_{ik} = \sum_{r=0}^{R-1} \frac{\tilde{c}_{ri}(p)}{x_k} f_r(x_k, p) \quad (i = 1, 2, \dots, I) \quad (1)$$

where  $R$ , for the moment, is an arbitrary natural number,  $f_r(x_k, p) = f_r\left(\frac{x_k - d(p)}{b(p)}\right)$  for a translated and deflated system, the  $\tilde{c}_{ri}(p)$  can be interpreted as coefficients, conditional on  $p$ , and the functions  $d(p)$  and  $b(p)$  are homogeneous of degree one.<sup>4</sup>

The rank of the demand system is the maximum number of dimensions spanned by the system's Engle curves. Equation (1) nests Gorman's (1981) rank 3 rationally derived system, Lewbel's (1989a) rank 4 rationally derived system, and Lewbel's (2003) translated deflated income system. At the level of specific applicable models it nests such well known ones as the translog (Christensen, Jorgenson, and Lau, 1975), AIDS (Deaton and Muellbauer, 1980), and QUAIDS (Banks, Blundell, and Lewbel, 1997). More generally, it is consistent with many studies in which expenditure systems have been found to be well approximated by finite (invariably low-order) log-income polynomials. In the case of rank 2 and rank 3 polynomials in logarithms of deflated expenditures, such as translog, AIDS and QUAIDS, equation (1) simplifies to

$$w_{ik} = \sum_{r=0}^{R-1} a_{ri}(p) (\ln x_k)^r \quad (i = 1, 2, \dots, I) \quad \text{for } R = 2, 3 \quad (2)$$

by dropping the translation term  $d(p)$  and by setting  $f_r(x_k, p) = \frac{x_k}{b(p)} \left[ \ln \left( \frac{x_k}{b(p)} \right) \right]^r$

and  $a_{ri}(p) = b(p) \sum_{j=r}^{R-1} \tilde{c}_{ji}(p) \binom{j}{j-r} (-\ln(b(p)))^{j-r}$ . Here  $R$  now represents the rank of the system.

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<sup>4</sup> To obtain this expenditure system we could begin with

$$q_{ik} = \sum_{r=0}^{R-1} \tilde{c}_{ri}(p) f_r(x_k, p) \quad (i = 1, 2, \dots, I) \quad \text{where } \tilde{c}_{ri}(p) = \frac{\tilde{c}_{ri}(p)}{p_i}.$$

Note that demographic, geographic, and other such household characteristics commonly included as additional variables in expenditure models can be accommodated in  $\tilde{c}_{0i}(p)$  and  $d(p)$ .



Reformulating  $\frac{f_r(x_k, p)}{x_k}$  in equation (1) as a Taylor series expansion in  $\ln x_k$  around  $\ln x_k = 0$  ( $x_k = 1$ ), and using the notation  $f_r$  to denote the function  $f_r(x_k, p)$ , results in

$$w_{ik} = \sum_{r=0}^{R-1} \tilde{c}_{ri}(p) \sum_{m=0}^{\infty} \frac{1}{m!} \left( \frac{\partial^m f_r}{\partial \ln x_k^m} + (-1)^m f_r \right) \Big|_{x_k=1} (\ln x_k)^m \quad (i = 1, 2, \dots, I),$$

with  $\frac{\partial^0 f_0}{\partial \ln x_k^0} = f_0$ . With regrouping of terms involving  $(\ln x_k)^m$ , this can be further simplified to

$$w_{ik} = \sum_{m=0}^{\infty} c_{mi}(p) (\ln x_k)^m \quad (i = 1, 2, \dots, I) \quad (3)$$

where  $c_{mi}(p) = \sum_{r=0}^{R-1} \tilde{c}_{ri}(p) \frac{1}{m!} \left( \frac{\partial^m f_r}{\partial \ln x_k^m} + (-1)^m f_r \right) \Big|_{x_k=1}$

The fact that equation (3) nests equation (2) can be seen by setting to zero all the derivatives of order higher than 2.

Elasticities (the focus of this paper) are invariant to scalar transformations of the units of measurement for income and prices. This allows us to simplify notation, without loss of generality (and with no implications for how a model might actually be estimated in practice), by introducing the normalization restrictions  $p_i = 1, \forall i$ , and  $\bar{x} = 1$

where  $\bar{x} = \sum_{k=1}^K \frac{x_k}{K}$ . Hereafter we write simply  $c_{mi}$ , if the context permits.

We now need an appropriate way of characterizing the income distribution. To that end we write  $X = \sum_{k=1}^K x_k$ ,  $y_k = \frac{x_k}{X}$ , and  $h_m = \sum_{k=1}^K y_k (\ln x_k - \ln \bar{x})^m$  ( $m = 0, 1, 2, \dots$ ). We can interpret  $h_m$  as a generalized measure of inequality (GMI) of order  $m$ . This is a

straightforward mathematical generalization of Theil's (1967) measure of inequality, which is obtained by setting  $m=1$ , and which was inspired by Shannon's (1948) measure of information entropy. An arbitrary income distribution can then be characterized by the sequence  $h_0, h_1, h_2$ , etc. (Note that  $h_0 = 1$ . Note too that  $h_m = 0$  for all  $m > 0$  when the distribution is uniform.) Invoking the normalization restriction  $\bar{x} = 1$  allows the simpler definition  $h_m = \sum_{k=1}^K y_k (\ln x_k)^m$ .

The GMIs provide a bridge from the micro specification of equation (3) to the corresponding specification at the aggregate level. Let  $X_i$  be aggregate expenditure on commodity  $i$  by all households and let  $W_i = \frac{X_i}{X} = \sum_{k=1}^K w_{ik} y_k$  be the aggregate expenditure share. Then

$$W_i = \sum_{m=0}^{\infty} c_{mi} \sum_{k=1}^K y_k (\ln x_k)^m = \sum_{m=0}^{\infty} c_{mi} h_m \quad (4)$$

For polynomials in logarithms of deflated expenditures defined in equation (2) for  $R = 2, 3$ , the aggregate expenditure share is

$$W_i = \sum_{r=0}^{R-1} a_{ri} \sum_{k=1}^K y_k (\ln x_k)^r = \sum_{r=0}^{R-1} a_{ri} h_r \quad (5)$$

Here, the  $W_i$  depend on GMIs up to order  $R-1$ ; the GMIs of order  $R$  and higher, which may be required to fully characterize some arbitrarily specified income distribution, are irrelevant for the determination of  $W_i$ . However, GMIs up to order  $2(R-1)$  are required for the determination of some elasticities and corresponding biases, as we show below.

#### 4. MEAN ELASTICITIES

Household  $k$  has a full (uncompensated) elasticity of demand for commodity  $i$  with respect to the price of commodity  $j$ ,  $\frac{\partial \ln q_{ik}}{\partial \ln p_j}$ , and a compensated elasticity

$$\left. \frac{\partial \ln q_{ik}}{\partial \ln p_j} \right|_{\bar{U}} = \frac{\partial \ln q_{ik}}{\partial \ln p_j} + w_{jk} \frac{\partial \ln q_{ik}}{\partial \ln x_k} \quad (6)$$

where  $\bar{U}$  indicates the constancy of utility. Now write  $Q_i = \sum_{k=1}^K q_{ik}$  for aggregate purchases of commodity  $i$ , all households combined,  $\phi_{lij}$  for the mean (same as aggregate) full price elasticity, and  $\eta_{lij}$  for the mean compensated price elasticity. (The significance of the 1 subscript will be apparent later.) We then have

$$\phi_{lij} = \frac{\partial \ln Q_i}{\partial \ln p_j} = \sum_{k=1}^K \frac{\partial \ln q_{ik}}{\partial \ln p_j} \cdot \frac{q_{ik}}{Q_i}$$

$$\text{and } \eta_{lij} = \phi_{lij} + \sum_{k=1}^K w_{jk} \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i} \quad (7)$$

where it is assumed (in the derivation of  $\eta_{lij}$ ) that households have a common utility function (but may of course be at different points on that function).

The expenditure elasticity for commodity  $i$  purchased by household  $k$  is  $\frac{\partial \ln q_{ik}}{\partial \ln x_k}$ . To derive a corresponding mean elasticity it is necessary to stipulate how a proportional increase in aggregate income is shared among households. The most common and straightforward assumption, and the one that we make here, is that the

proportional change is the same for all households, so that  $\frac{\partial \ln x_k}{\partial \ln X} = 1$  for all  $k$ .<sup>5</sup>

Writing  $\varepsilon_{li}$  for the mean expenditure elasticity we then have

$$\varepsilon_{li} = \frac{\partial Q_i}{\partial X} \cdot \frac{X}{Q_i} = \sum_{k=1}^K \frac{\partial q_{ik}}{\partial x_k} \cdot \frac{x_k}{q_{ik}} \cdot \frac{q_{ik}}{Q_i} \cdot \frac{X}{x_k} \cdot \frac{\partial x_k}{\partial X} = \sum_{k=1}^K \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i} \quad (8)$$

## 5. BIASES IN THE USE OF MICRO DATA

Given an appropriate set of data for individual households and an expenditure system defined by equation (3), price and expenditure elasticities can be calculated directly, whatever the distribution of income. These elasticities are the correct ones for evaluating aggregate effects. Elasticities at the mean of the income distribution can also be calculated, either for their own value or as (biased) approximations to the mean elasticities. We present below the results for the corresponding biases, and later, results in the form of two theorems. (All proofs are provided in Appendix A. See again Figure 1, which provides a schematic illustration of the relationships among the biases, and the related discussion in section 2.)

There are no constraints on the rank of a demand system with regard to the existence of expenditure or full price elasticities.<sup>6</sup> The derivation (found in Appendix A) and the interpretation of the biases do not require placing constraints on the maximum rank of the demand system. However, the existence of compensated price elasticities (under the assumption of rationality or, in other words, consistency with Gorman's, 1981 demand system) requires the rank to be at most four (Lewbel, 1989a). Thus while the following theorems relate to systems of arbitrary rank they have meaning for compensated price elasticities only for systems up to rank four.

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<sup>5</sup> This assumption is consistent with what Lewbel (1989b, 1990) calls "mean scaling."

<sup>6</sup> Lau (1977) develops a theory of exact aggregation for systems of any rank, where aggregate demand can be expressed in terms of index functions such as the GMIs that we are using.

*Theorem 1:* Based on the underlying micro demand model specified in equation (3) and the corresponding aggregate model of equation (4), the bias in interpreting an elasticity at mean income (EM) as the mean elasticity (ME), as in Figure 1, is as follows:

(i) for the full price elasticity

$$\phi_{2ij} - \phi_{1ij} = \frac{\left( \frac{\partial c_{0i}}{\partial \ln p_j} \right) \sum_{m=0}^{\infty} c_{mi} h_m - \left( \sum_{m=0}^{\infty} \left( \frac{\partial c_{mi}}{\partial \ln p_j} \right) h_m \right) c_{0i}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} .$$

(ii) for the compensated price elasticity

$$\eta_{2ij} - \eta_{1ij} = \frac{\left\{ \left( \left( \frac{\partial c_{0i}}{\partial \ln p_j} + c_{0j} c_{0i} \right) + c_{0j} c_{1i} \right) \sum_{m=0}^{\infty} c_{mi} h_m - \left( \sum_{m=0}^{\infty} \left( \frac{\partial c_{mi}}{\partial \ln p_j} \right) h_m + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} m c_{mi} c_{nj} h_{m+n-1} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mi} c_{nj} h_{m+n} \right) c_{0i} \right\}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} .$$

(iii) for the bias for the expenditure elasticity

$$\varepsilon_{2i} - \varepsilon_{1i} = \frac{c_{1i} \sum_{m=0}^{\infty} c_{mi} h_m - c_{0i} \sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} .$$

*Special Case:* For ranks 2 and 3 ( $R=2,3$ , the ranks of the most commonly used consumer demand functions), with polynomials in logarithms of deflated expenditures, as defined in equation (2), the bias in interpreting an elasticity at mean income as the mean elasticity is as follows:

(a) for the full price elasticity

$$\phi_{2ij} - \phi_{1ij} = \frac{\left( \frac{\partial a_{0i}}{\partial \ln p_j} \right) \sum_{r=0}^{R-1} a_{ri} h_r - \left( \sum_{r=0}^{R-1} \left( \frac{\partial a_{ri}}{\partial \ln p_j} \right) h_r \right) a_{0i}}{a_{0i} \sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order  $R-1$ ;

(b) for the compensated price elasticity

$$\eta_{2ij} - \eta_{1ij} = \frac{\left\{ \left( \left( \frac{\partial a_{0i}}{\partial \ln p_j} + a_{0j} a_{0i} \right) + a_{0j} a_{1i} \right) \sum_{r=0}^{R-1} a_{ri} h_r - \left( \sum_{r=0}^{R-1} \left( \frac{\partial a_{ri}}{\partial \ln p_j} \right) h_r + \sum_{r=1}^{R-1} \sum_{s=0}^{R-1} r a_{ri} a_{sj} h_{r+s-1} + \sum_{r=0}^{R-1} \sum_{s=0}^{R-1} a_{ri} a_{sj} h_{r+s} \right) a_{0i} \right\}}{a_{0i} \sum_{r=0}^{R-1} a_{ri} h_r} \text{ and thus is a}$$

function of GMIs up to order  $2(R-1)$ ;

(c) for the expenditure elasticity

$$\varepsilon_{2i} - \varepsilon_{1i} = \frac{a_{1i} \sum_{r=0}^{R-1} a_{ri} h_r - a_{0i} \sum_{r=1}^{R-1} r a_{ri} h_{r-1}}{a_{0i} \sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order  $R-1$ .

## 6. BIASES IN THE USE OF AGGREGATE DATA

Micro data are often not available, or not suitable, for the estimation of particular models and elasticities, and aggregate data may have to be used (see footnote 2), thus introducing the possibility of aggregation bias. The common practice is to assume that the micro model holds at the aggregate level, which in general it does not – to assume, that is, that equation (3) holds with  $w_{ik}$  and  $x_k$  replaced by their aggregate counterparts,  $W_i$  and  $\bar{x}$ . On that basis the variant of equation (3) employed when using aggregate data is

$$W_i = \sum_{m=0}^{\infty} \hat{c}_{mi}(p)(\ln \bar{x})^m \quad (i = 1, 2, \dots, I) \quad (9)$$

where  $\hat{c}_{mi}(p) = \sum_{n=m}^{\infty} e_{mn} c_{ni}(p) h_{n-m}$ ,  $e_{0m} = e_{mm} = 1$  for  $m = 0, 1, 2, \dots$ ,

$e_{mm} = e_{m,n-1} + e_{m-1,n-1}$  for  $m = 1, 2, \dots, n = m+1, m+2, \dots$ .

The associated full price, compensated price, and expenditure elasticities for this model are obtained as

$$\phi_{3ij} = \frac{\partial \ln Q_i}{\partial \ln p_j}, \quad \eta_{3ij} = \left. \frac{\partial \ln Q_i}{\partial \ln p_j} \right|_U, \quad \text{and} \quad \varepsilon_{3i} = \frac{\partial \ln Q_i}{\partial \ln X},$$

$$\text{where } \eta_{3ij} = \phi_{3ij} + W_j \varepsilon_{3i}. \quad (10)$$

If the elasticities derived at mean income using aggregate data are interpreted as if they were the true mean elasticities, the biases are as given in Theorem 2. If on the other hand the results are interpreted as elasticities at mean income the biases are as given in Theorem 3.<sup>7</sup>

*Theorem 2:* The bias in interpreting an elasticity at mean income using aggregate data (AM) (based on equation (9)), as the mean elasticity (ME), as in Figure 1, is as follows:

(i) for the full price elasticity

$$\phi_{3ij} - \phi_{1ij} = \frac{- \sum_{m=1}^{\infty} \left( \frac{\partial c_{mi}}{\partial \ln p_j} \right) h_m - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m}.$$

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<sup>7</sup> Theorem 3 is in the spirit of the research dealing with the biases in using aggregate data to estimate micro structural price and income parameters (Blundell, Pashardes, and Weber, 1993, Blundell, Meghir, and Weber, 1993), and biases in using aggregate macro-based elasticities to estimate micro elasticities at mean income (Denton and Mountain, 2001, 2004).

(ii) for the compensated price elasticity

$$\eta_{3ij} - \eta_{1ij} = \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mi} c_{nj} [h_m h_n - h_{m+n}] - \sum_{m=1}^{\infty} \left( \frac{\partial c_{mi}}{\partial \ln p_j} \right) h_m - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m+n-1}}{\sum_{m=0}^{\infty} c_{mi} h_m}$$

(iii) for the expenditure elasticity

$$\varepsilon_{3i} - \varepsilon_{1i} = 0$$

and therefore the AM elasticity possesses no bias in estimating the ME elasticity for every income distribution.<sup>8</sup>

*Special Case:* For ranks 2 and 3, with polynomials in logarithms of deflated expenditures, as defined in equation (2), the bias in interpreting an elasticity at mean income using aggregate data as the mean elasticity is as follows:

(a) for the full price elasticity

$$\phi_{3ij} - \phi_{1ij} = \frac{-\sum_{r=1}^{R-1} \left( \frac{\partial a_{ri}}{\partial \ln p_j} \right) h_r - \sum_{r=1}^{R-1} \sum_{s=1}^{R-1} r a_{ri} a_{sj} h_{r-1} h_s}{\sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order  $R-1$ ;

(b) for the compensated price elasticity

$$\eta_{3ij} - \eta_{1ij} = \frac{\sum_{r=1}^{R-1} \sum_{s=1}^{R-1} a_{ri} a_{sj} [h_r h_s - h_{r+s}] - \sum_{r=1}^{R-1} \left( \frac{\partial a_{ri}}{\partial \ln p_j} \right) h_r - \sum_{r=1}^{R-1} \sum_{s=1}^{R-1} r a_{ri} a_{sj} h_{r+s-1}}{\sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order  $2(R-1)$ ;

(c) for the expenditure elasticity

$$\varepsilon_{3i} - \varepsilon_{1i} = 0$$

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<sup>8</sup> Among the biases calculated, this is the only one that is identically zero for all functional forms of demand systems.



*Theorem 3:* The bias in interpreting an elasticity derived at mean income using aggregate data (AM) as the true elasticity at mean income (EM), as in Figure 1, is equal to the difference between the biases of Theorem 2 and Theorem 1. This bias is therefore:

- (i) for the full price elasticity  $\phi_{3ij} - \phi_{2ij} = (\phi_{3ij} - \phi_{1ij}) - (\phi_{2ij} - \phi_{1ij})$
- (ii) for the compensated elasticity  $\eta_{3ij} - \eta_{2ij} = (\eta_{3ij} - \eta_{1ij}) - (\eta_{2ij} - \eta_{1ij})$
- (iii) for the expenditure elasticity  $\varepsilon_{3i} - \varepsilon_{2i} = (\varepsilon_{3i} - \varepsilon_{1i}) - (\varepsilon_{2i} - \varepsilon_{1i})$

*Special Case:* For ranks 2 and 3, with polynomials in logarithms of deflated expenditures, as defined in equation (2), the bias in interpreting an elasticity at mean income using aggregate data (AM) as the true elasticity at mean income (EM) is calculated using (i), (ii) or (iii) in Theorem 3. All of these biases are functions of GMIs up to order  $R-1$ .

All of the biases in Theorems 2 and 3 are (in general) nonzero, with the notable exception of the expenditure elasticity bias in Theorem 2, where aggregate data are used to estimate the mean elasticity, and the bias is zero. The notion of a “representative consumer” is often invoked to justify the use of aggregate data. For the expenditure elasticity the representative consumer turns out in fact to be a household with mean elasticity, whatever the rank of the system and the distribution of income. For the price elasticities, though, that is not the case.

## 7. THE SET-UP FOR SIMULATION WITH ALTERNATIVE MODELS

Four models of demand systems that are familiar in the literature are the translog (TLOG), the linear Almost Ideal Demand System (AIDS), the quadratic extension of the linear system (QUAIDS), and Lewbel’s rank 4 demand system, which we shall refer to as L4. TLOG and AIDS are rank 2 systems, QUAIDS is a rank 3 system. We use calibrated versions of these four models to simulate the biases discussed above and explore their possible magnitudes.

The TLOG model (Christensen, Jorgenson, and Lau, 1975) is defined at the micro level by the equation<sup>9</sup>

$$w_{ik} = \frac{\alpha_i^* + \sum_{j=1}^I \gamma_{ij}^* \ln p_j - \sum_{j=1}^I \gamma_{ij}^* \ln x_k}{-1 + \sum_{i=1}^I \sum_{j=1}^I \gamma_{ij}^* \ln p_j} . \quad (11)$$

Under normalization of prices and income this becomes  $w_{ik} = -\alpha_i^*$  and the corresponding aggregate form, consistent with equation (5), is  $W_i = -\left(\alpha_i^* - \sum_{j=1}^I \gamma_{ij}^* h_1\right)$ .

The QUAIDS model (Banks, Blundell, and Lewbel, 1997) is defined by

$$w_{ik} = \alpha_i + \sum_{j=1}^I \gamma_{ij} \ln p_j + \beta_i \ln(x_k / b) + \lambda_i (\ln(x_k / b))^2 / B \quad (12)$$

with  $\ln b = \sum_{i=1}^I \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^I \gamma_{ij} (\ln p_i)(\ln p_j)$  and  $\ln B = \sum_{i=1}^I \beta_i \ln p_i$ .

Under normalization this becomes  $w_i = \alpha_i$ , with corresponding aggregate form

$$W_i = \alpha_i + \beta_i h_1 + \lambda_i h_2 .$$

The linear AIDS model (Deaton and Muellbauer, 1980) is obtained by setting  $\lambda_i = 0, \forall i$ , in equation (12), and omitting  $\ln B$ .  $W_i$  is then equal to  $\alpha_i + \beta_i h_1$ , under normalization.

The L4 model (Lewbel, 2003) is defined by

$$w_{ik} = \frac{\tau_i d}{x_k} + \left(1 - \frac{d}{x_k}\right) \left( \alpha_i + \sum_{j=1}^I \gamma_{ij} \ln p_j + \beta_i [\ln(x_k - d) - \ln b] + \lambda_i [\ln(x_k - d) - \ln b]^2 / B \right) \quad (13)$$

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<sup>9</sup> This formulation of the translog model is also found in Jorgenson and Slesnick (1984).

with  $\ln b = \alpha_0 + \sum_{i=1}^I \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^I \gamma_{ij} (\ln p_i)(\ln p_j)$ ,  $d = \rho_0 \prod_{i=1}^I p_i^{\tau_i}$  and

$\ln B = \sum_{i=1}^I \beta_i \ln p_i$ .<sup>10</sup> Note that L4 nests QUAIDS ( $\rho_0 = 0$  and  $\tau_i = 0$  for  $i = 1, 2, \dots, I$ )

and hence AIDS ( $\lambda_i = 0$ , additionally, for  $i = 1, 2, \dots, I$ ).

With normalization, equation (13) becomes  $w_i = \tau_i \rho_0 + (1 - \rho_0) \alpha_i$ .<sup>11</sup> In the form of equation (3), the Taylor series expansion of equation (13) is

$$w_{ik} = \sum_{m=0}^3 c_{mi}(p) (\ln x_k)^m + \text{terms of higher order in } m \quad (i = 1, 2, \dots, I) \quad (14)$$

with the  $c_{mi}$  being the Taylor series coefficients (see Denton and Mountain, 2011b for more detail).

The formulas for the biases in the elasticities derived from these four models, corresponding to Theorems 1, 2 and 3, are displayed in Tables B1, B2 and B3 of Appendix B.<sup>12</sup> For convenience, we use again the following symbols in the tables (as in Figure 1): EM – elasticity at the mean of the income distribution; ME – mean elasticity based (correctly) on micro data; AM – elasticity at mean income based on aggregate data.

All of the biases derived in the tables are (in general) nonzero, with the exception of the expenditure elasticities in Table B2. Moreover, for the TLOG and AIDS models, as shown in Table B1, the EM expenditure elasticity is always greater than the ME expenditure elasticity (a positive bias) and as shown in Table B3 the AM expenditure elasticity is always less than the EM expenditure elasticity (a negative bias) for  $h_1 > 0$ .<sup>13</sup>

<sup>10</sup> Two small typos appear in Lewbel's (2003) original paper. The corrected version of the model can be found in Lewbel (2004). The demand system in equation (13) is the correct version.

<sup>11</sup> Without loss of generality, part of the normalization for the L4 demand system is  $\alpha_0 = \ln(1 - \rho_0)$ .

<sup>12</sup> The formulas for the biases in comparing the AM and EM elasticities in Table B3 for AIDS can be found also in Denton and Mountain, 2001, and for QUAIDS in Denton and Mountain, 2004.

<sup>13</sup> For a wide range of income (expenditure) inequalities observed in OECD countries, calculations by Denton and Mountain (2001, 2011a) show that  $h_1$  is always positive.

With respect to the TLOG and AIDS models with  $h_1 > 0$  (as shown in Table B2) for full own-price elasticities and full cross-price elasticities, where both goods are luxuries ( $\varepsilon_{2i}, \varepsilon_{2j} > 1$ ), or where both goods are necessities ( $\varepsilon_{2i}, \varepsilon_{2j} < 1$ ), the bias in using an AM price elasticity to estimate an ME price elasticity is negative ( $\phi_{3ij} < \phi_{1ij}$ ). On the other hand, for the TLOG and AIDS models with  $h_1 > 0$ , for full cross-price elasticities where one of the goods is a luxury and the other a necessity, the bias in using an AM cross-price elasticity to estimate an ME cross-price elasticity is positive ( $\phi_{3ij} > \phi_{1ij}$ ). In all of these situations, the larger is the expenditure inequality (the larger is  $h_1$ ), the larger is the absolute value of the bias.

## 8. CALIBRATION

To explore the quantitative implications of the biases derived in the previous section we calibrate each of the four models based on realistic values drawn from the empirical literature, and calibrate also the income distribution parameters.

Values for the model micro parameters are drawn from econometric estimates in Blundell, Pashardes, and Weber (1993). These estimates are based on monthly time series of repeated cross-sections from the British Family Expenditure Survey covering some 4000 households over 15 years. We take mean  $w_i$  values from table A1 of that paper for the six expenditure categories that the authors identify for estimation.<sup>14</sup> Values for the six micro expenditure and own-price compensated and full elasticities are based on the Blundell et al. generalized method of moments estimates in their tables 3A and 3B. For the TLOG, AIDS and QUAIDS models, the calculation of parameter values corresponding to the micro elasticities is straightforward. For the L4 model, additional parameters ( $\tau_i, i = 1, 2, \dots, 6; \rho_0$ ) must be chosen before calculation of the remaining ones. Because the  $\rho_0$  parameter can be interpreted as a committed expenditure component (with  $p_i = 1$ , under normalization), we selected  $\rho_0 = 0.2$  after consulting a number of related estimates in the literature that use either the L4 model or linear or quadratic

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<sup>14</sup> A seventh category was dropped by the authors because of the singularity of the expenditure system.

expenditure models.<sup>15</sup> The values that we assign to the model parameters are provided in our Appendix C, Table C1, retaining in that table and others the names of the expenditure categories used by Blundell et al. (food, alcohol, fuel, clothing, transport, and services).

The biases are the result of interactions between the micro parameters and the parameters of the income distribution. In calibrating the income distribution parameters we draw on data and analysis provided in O’Higgins, Schmaus, and Stephenson (1989). The data used in that article are from the Luxembourg Income Study (LIS) data base and relate to seven OECD countries: Canada, the United States, the United Kingdom, Germany, Sweden, Norway, and Israel. Of particular interest to us, for present purposes, are countries with substantially different degrees of distributional inequality, and with that in mind we have chosen two: Sweden, with the lowest degree of inequality, and Germany, with the highest degree, as indicated by their Gini coefficients.<sup>16</sup> O’Higgins et al. present and analyse the LIS income data in various ways; we have chosen family net (after-tax) income as most suitable for our purposes.

The Gini coefficient for Sweden is 29.2, in percentage form, and the corresponding coefficient for Germany is 40.9. A characteristic of the German income data noted by O’Higgins et al. is that the data include a relatively large number of zero or negative incomes; if those are eliminated the authors report that the Gini coefficient declines slightly to 38.9, still much higher than the Swedish figure, and still the highest of the seven countries. A possible correlate in the comparison is a higher proportion of income from self-employment in the German data since self-employment is more likely to generate zero or negative incomes. Self-employment income as a proportion of average gross income is 16.7 percent in the German data, whereas in the Swedish data it is only 3.7 percent. The percentages for the other countries, followed by the Gini coefficients, are as follows: Norway 11.1, 31.1; Israel 16.8, 33.8; Canada 5.4, 34.8; United Kingdom 4.5, 34.3; United States, 6.7, 37.0. More recent data might well show a different ranking among the countries, of course, but the data provided in the study serve our purposes.

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<sup>15</sup> See, for example, Andrikopoulos, Brox, and Gamaletsos, 1984, Howe, Pollak, and Wales, 1979, Lewbel, 2003, Lewis and Andrews, 1989, Pollak and Wales, 1978, Wales, 1971.

<sup>16</sup> The data collected and organized by the LIS for these countries originated with the Swedish Income Distribution and Level of Living Survey and the German Transfer Survey.

Further evidence of the distributional differences between Sweden and Germany is provided by the proportions of cash benefits (transfers) in the two countries. In the Swedish data the benefits amount to 29.2 percent of gross income; in the German data they amount to only 16.5 percent. Concomitantly, income tax is 28.5 percent of gross income in the Swedish data, only 14.8 percent in the German data.

The income distribution parameters required for the simulations are the  $h_m$  parameters. Simulations with AIDS and the Translog model require only  $h_1$ ; simulations with QUAIDS require  $h_1$  and  $h_2$ . In theory, simulations with L4 require all values of  $h_m$  for  $m > 0$ . However, as a practical matter we use  $h_m$  for  $m = 1, 2, 3, 4$  as an approximation for this model.

The values of the  $h_m$  parameters that we use in calibration are shown in Table C2 of Appendix C for  $m = 1, 2, 3, 4$ . Values are shown for all seven of the OECD countries noted above. As seen in the table, Sweden has the lowest values of  $h_1$  and  $h_2$ , Germany the highest. The value of  $h_2$  is necessarily positive. In theory, the value of  $h_1$  could be negative but all values in the table are positive, and that is true over a much wider range of OECD countries than is shown in the table (see Denton and Mountain, 2001, 2011a).

## 9. SIMULATION RESULTS

With the underlying micro parameters, we then calculated the biases for countries with the least and greatest inequality of income distribution, Sweden and Germany. Tables 1, 2, 3 and 4 report the corresponding mean elasticities (ME), the micro elasticity calculated at the mean for the income distribution (EM), and the elasticity at mean income based on aggregate data (AM) for all four models. Percentage biases in estimating ME with either EM or AM elasticity are calculated. This is done for full own-price elasticities, compensated own-price elasticities and expenditure elasticities.

There are both similarities and variations across all four models. The main conclusions are as follows. The greater the income inequality, the greater is the bias. The biases for Sweden are generally smaller than those for Germany. For example, for food and the L4 model, the EM% biases and AM% biases for full and compensated price elasticities are more than twice as large for Germany than for Sweden. Furthermore, the greater the departure of the expenditure elasticity from one, the greater is the bias. For

example, for food and alcohol, with EMs of 0.61 and 2.29, respectively, the biases for the TLOG and AIDS models are relatively large. This is in contrast with the results for clothing, with an EM of 0.92. As we consider the models of higher rank, QUAIDS and L4, the biases tend to get larger. Here, elasticities at mean income, whether using micro or macro data, are not capturing the curvature of the Engle curves, and hence of the mean elasticities corresponding to higher rank models. For the clothing expenditure elasticity for Germany, for example, EM overestimates ME by 19.58 % and 25.77 % for the QUAIDS and L4 models, respectively. Generally, although not always, in estimating the ME elasticity, the AM elasticity does a better job than the EM. However, the percentage biases associated with AM for compensated elasticities are considerably higher (as high as 15.55 % for L4) than those for the full price elasticities. This is partly due to the zero AM bias. In terms of absolute size, the expenditure elasticity bias tends to be greater than the full price and compensated price elasticity biases.

For the L4 model we tried different values of  $\rho_0$  to investigate the sensitivity of the results to the differences in that parameter. When we changed the value from 0.2 to 0.5 we found the biases in the price elasticities to be only slightly larger. However, we did find some much larger biases involving expenditure elasticities (e.g.,  $EM - ME$  differences of 0.113 and 0.235 for food in Sweden and Germany with  $\rho_0 = 0.5$ , compared with 0.055 and 0.111 for  $\rho_0 = 0.2$ ).

## 10. CONCLUSION

We began this paper by noting that consumer-related policy decisions frequently require the evaluation of aggregate responses, often in the form of mean price or expenditure elasticities. As we noted, quantifying the aggregate response of commodity taxes (gasoline, cigarette), carbon taxes, trade tariffs or changes in aggregate income (resulting from changes in rates of taxation or income subsidies) are examples of policy applications that require aggregate elasticities. Such elasticities can be derived by calculations based on a properly specified model fitted to micro data, but in practice that is seldom done. They can be approximated from a model fitted to aggregate data, but the approximation introduces the possibility of aggregation bias in the calculation of price

elasticities, though interestingly not in the calculation of expenditure elasticities. We have provided in this paper formulas and a range of numerical values for the correct calculation of mean elasticities – expenditure and both full and compensated price elasticities – and the corresponding biases when incorrect formulas are used. The correct formulas and the numerical biases depend in general on the type of data (micro or aggregate), the type of model being estimated, the rank of the model, and the characteristics of the income distribution. Such biases should caution analysts concerned with policies at the macro level to take care not to be misled in using elasticities calculated at mean income rather than the more appropriate mean elasticities. As we have seen, the differences can be substantial.

We have quantified the range of biases for familiar demand systems. The empirical results are robust in that the estimated biases are of the same order of magnitude, regardless of the functional form. Whether we use AM or EM elasticities to estimate the ME elasticity, the biases increase as the degree of income inequality grows and as the underlying expenditure elasticities depart from one. They increase also as the rank of the model increases. Generally, the AM elasticity performs a better job than the EM elasticity in representing ME, although direct estimation of ME itself, where data permit, is of course the preferred option.



Figure 1: Relationships Among Elasticities and Biases

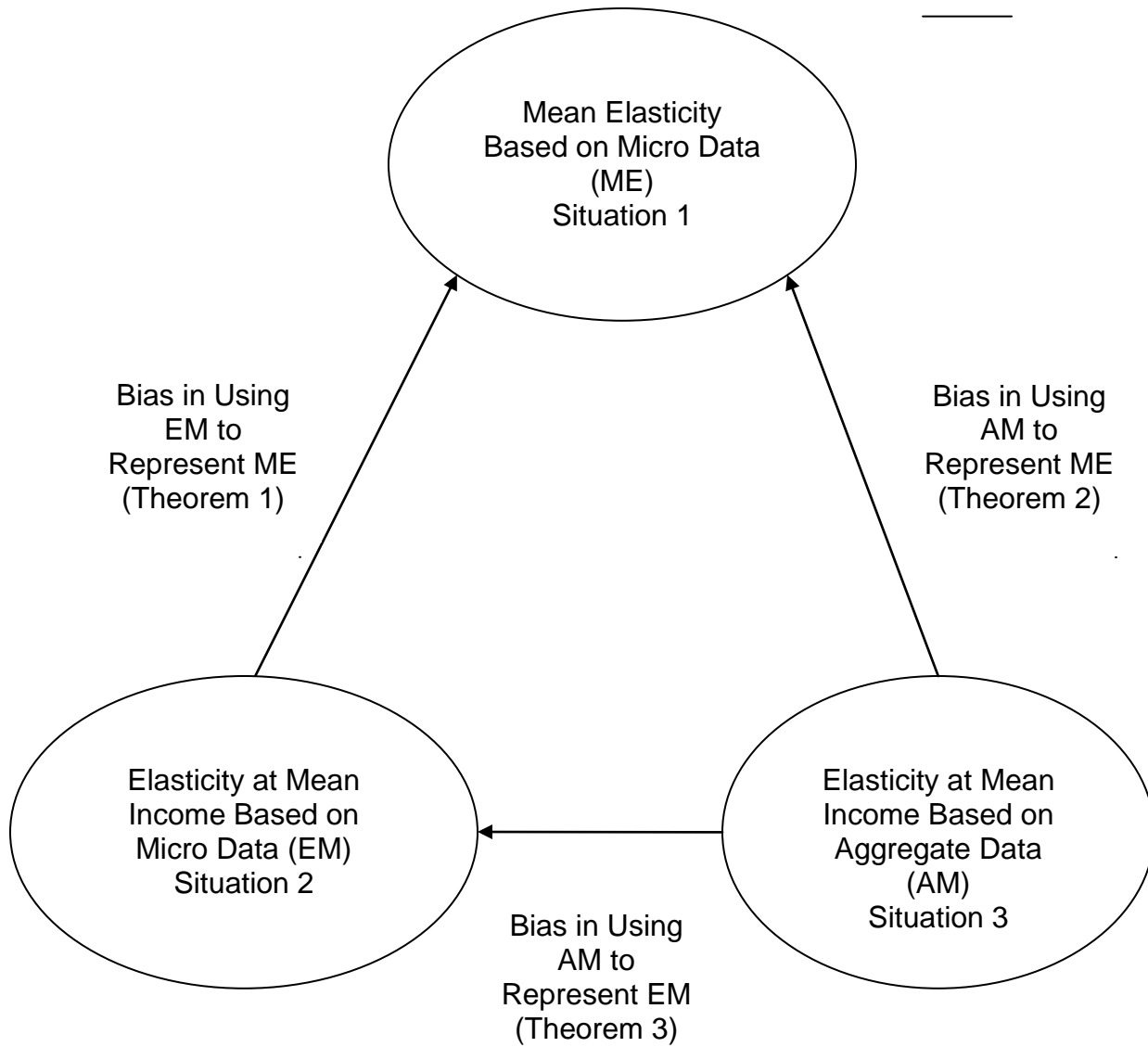


TABLE 1: SIMULATED ELASTICITY BIASES FOR TLOG DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.535	-0.507	-1.627	-1.551	-0.507	-0.498	-0.618	-0.615	-0.702	-0.708	-0.736	-0.745
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.542	-0.521	-1.639	-1.571	-0.508	-0.499	-0.618	-0.615	-0.703	-0.709	-0.738	-0.750
EM % Bias	5.40	11.07	6.99	12.21	1.96	3.78	0.62	1.17	-0.92	-1.66	-1.58	-2.78
AM % Bias	1.29	2.64	0.76	1.33	0.05	0.10	0.01	0.02	0.12	0.22	0.38	0.68
Compensated Own-Price												
ME	-0.325	-0.301	-1.432	-1.334	-0.441	-0.433	-0.527	-0.524	-0.480	-0.481	-0.550	-0.549
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.345	-0.338	-1.468	-1.391	-0.442	-0.435	-0.527	-0.525	-0.483	-0.485	-0.558	-0.563
EM % Bias	7.61	16.09	10.31	18.47	2.01	3.87	0.57	1.07	-0.09	-0.15	0.04	0.09
AM % Bias	6.02	12.26	2.46	4.27	0.17	0.31	0.04	0.08	0.51	0.91	1.47	2.54
Expenditure												
ME	0.590	0.572	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.590	0.572	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM % Bias	3.33	6.69	8.36	14.74	0.38	0.73	0.09	0.16	0.40	0.74	1.65	2.99
AM % Bias	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: EM % Bias is calculated as  $[EM-ME]/ME$  and the AM % Bias is calculated as  $[AM-ME]/ME$ . See Figure 1 for definition of ME, EM and AM.

TABLE 2: SIMULATED ELASTICITY BIASES FOR AIDS DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.542	-0.521	-1.639	-1.571	-0.508	-0.499	-0.618	-0.615	-0.703	-0.709	-0.738	-0.750
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.548	-0.534	-1.651	-1.592	-0.508	-0.499	-0.618	-0.615	-0.704	-0.711	-0.741	-0.755
EM % Bias	4.06	8.22	6.19	10.74	1.96	3.68	0.61	1.15	-1.04	-1.88	-1.96	-3.44
AM % Bias	1.27	2.57	0.75	1.31	0.05	0.10	0.01	0.02	0.12	0.22	0.38	0.67
Compensated Own-Price												
ME	-0.332	-0.315	-1.445	-1.354	-0.441	-0.434	-0.527	-0.525	-0.481	-0.482	-0.553	-0.555
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.352	-0.352	-1.480	-1.411	-0.442	-0.435	-0.527	-0.525	-0.484	-0.487	-0.561	-0.568
EM % Bias	5.39	11.165	9.36	16.67	1.95	3.76	0.55	1.05	-0.27	-0.48	-0.47	-0.82
AM % Bias	5.89	11.74	2.43	4.20	0.17	0.31	0.04	0.08	0.51	0.90	1.46	2.51
Expenditure												
ME	0.590	0.572	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.590	0.590	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM % Bias	3.33	6.69	8.36	14.74	0.38	0.73	0.09	0.16	0.40	0.74	1.65	2.99
AM % Bias	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: EM % Bias is calculated as  $[EM-ME]/ME$  and the AM % Bias is calculated as  $[AM-ME]/ME$ . See Figure 1 for definition of ME, EM and AM.

TABLE 3: SIMULATED ELASTICITY BIASES FOR QUAIDS DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.538	-0.513	-1.642	-1.576	-0.565	-0.601	-0.586	-0.549	-0.713	-0.727	-0.715	-0.706
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.545	-0.527	-1.654	-1.596	-0.566	-0.604	-0.587	-0.551	-0.714	-0.729	-0.718	-0.712
EM % Bias	4.83	9.92	6.00	10.45	-8.46	-13.94	6.15	13.22	-2.42	-4.28	1.30	2.52
AM % Bias	1.31	2.65	0.75	1.29	0.18	0.48	0.10	0.37	0.14	0.28	0.42	0.78
Compensated Own-Price												
ME	-0.331	-0.311	-1.449	-1.361	-0.483	-0.503	-0.506	-0.479	-0.483	-0.484	-0.547	-0.543
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.352	-0.352	-1.484	-1.417	-0.482	-0.506	-0.508	-0.485	-0.487	-0.491	-0.550	-0.550
EM % Bias	5.88	12.54	9.01	16.05	-6.79	-10.58	4.67	10.63	-0.56	-0.82	0.60	1.23
AM % Bias	6.41	11.56	2.37	3.93	-0.10	0.50	0.37	1.22	0.79	1.47	0.64	1.23
Expenditure												
ME	0.582	0.555	2.114	1.996	0.958	1.044	0.845	0.769	1.211	1.220	1.394	1.347
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.582	0.555	2.114	1.996	0.958	1.044	0.845	0.769	1.211	1.220	1.394	1.347
EM % Bias	4.81	9.88	8.33	14.74	-12.29	-19.55	8.91	19.58	-0.91	-1.62	4.00	7.66
AM % Bias	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: EM % Bias is calculated as  $[EM-ME]/ME$  and the AM % Bias is calculated as  $[AM-ME]/ME$ . See Figure 1 for definition of ME, EM and AM.

TABLE 4: SIMULATED ELASTICITY BIASES FOR L4 DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.532	-0.496	-1.612	-1.534	-0.558	-0.588	-0.578	-0.536	-0.717	-0.733	-0.714	-0.711
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.536	-0.505	-1.629	-1.563	-0.557	-0.588	-0.578	-0.537	-0.719	-0.738	-0.718	-0.718
EM % Bias	5.95	13.70	7.96	13.43	-7.37	-12.09	7.56	16.10	-2.86	-4.99	1.41	1.89
AM % Bias	0.85	1.98	1.05	1.87	-0.16	-0.10	0.02	0.22	0.39	0.72	0.61	1.09
Compensated Own-Price												
ME	-0.332	-0.306	-1.406	-1.296	-0.478	-0.494	-0.501	-0.469	-0.480	-0.478	-0.543	-0.542
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.356	-0.354	-1.451	-1.370	-0.476	-0.495	-0.503	-0.476	-0.487	-0.491	-0.548	-0.552
EM % Bias	5.27	14.27	12.39	21.87	-5.90	-9.08	5.77	12.94	-0.04	0.34	1.32	1.45
AM % Bias	7.09	15.55	3.20	5.65	-0.46	-0.23	0.34	1.36	1.33	2.55	1.05	1.88
Expenditure												
ME	0.555	0.499	2.110	1.996	0.944	1.023	0.827	0.732	1.221	1.238	1.400	1.358
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.555	0.499	2.110	1.996	0.958	1.023	0.827	0.732	1.221	1.258	1.400	1.358
EM % Bias	9.98	22.16	8.55	14.71	-11.04	-17.85	11.21	25.77	-1.75	-3.06	3.56	6.75
AM % Bias	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: EM % Bias is calculated as  $[EM-ME]/ME$  and the AM % Bias is calculated as  $[AM-ME]/ME$ . See Figure 1 for definition of ME, EM and AM.

## Appendix A – Theorem Proofs

### Proof of Theorem 1:

Step 1:

(i) Using equations (3) and (4) (and using the normalization restrictions, both here and subsequently), the *mean full price elasticity* ( $\phi_{lij}$ ) is given by

$$\begin{aligned}
 \phi_{lij} &= \sum_{k=1}^K \frac{\partial \ln q_{ik}}{\partial \ln p_j} \frac{q_{ik}}{Q_i} \\
 &= \sum_{k=1}^K \left( -\delta_{ij} + \frac{\frac{\partial w_{ik}}{\partial \ln p_j}}{w_{ik}} \right) \frac{q_{ik}}{Q_i} \\
 &= \sum_{k=1}^K \left( -\delta_{ij} + \frac{\sum_{m=0}^{\infty} \left[ \frac{\partial c_{mi}}{\partial \ln p_j} \right] (\ln x_k)^m}{w_{ik}} \right) \frac{q_{ik}}{Q_i} \\
 &= -\delta_{ij} + \sum_{k=1}^K \left( \left( \frac{\sum_{m=0}^{\infty} \left[ \frac{\partial c_{mi}}{\partial \ln p_j} \right] (\ln x_k)^m}{W_i} \right) \left( \frac{x_k}{X} \right) \right) \\
 &= -\delta_{ij} + \frac{\sum_{m=0}^{\infty} \left[ \frac{\partial c_{mi}}{\partial \ln p_j} \right] h_m}{\sum_{m=0}^{\infty} c_{mi} h_m} \tag{A1}
 \end{aligned}$$

(ii) With respect to the second term of equation (7), using equations (3) and (4) again,

$$\begin{aligned}
\sum_{k=1}^K w_{jk} \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i} &= \sum_{k=1}^K w_{jk} \left[ \frac{\partial \ln \left( \frac{w_{ik} x_k}{p_i} \right)}{\partial \ln x_k} \frac{w_{ik}}{W_i} \frac{x_k}{X} \right] \\
&= \sum_{k=1}^K w_{jk} \left[ \left( 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1}}{w_{ik}} \right) \frac{w_{ik}}{W_i} \frac{x_k}{X} \right] \\
&= \sum_{k=1}^K \frac{w_{jk} w_{ik}}{W_i} \frac{x_k}{X} + \frac{\sum_{k=1}^K w_{jk} \sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1} \frac{x_k}{X}}{W_i} \\
&= \frac{\sum_{k=1}^K \sum_{n=0}^{\infty} c_{nj} (\ln x_k)^n \sum_{m=0}^{\infty} c_{mi} (\ln x_k)^m \frac{x_k}{X} + \sum_{k=1}^K \sum_{n=0}^{\infty} c_{nj} (\ln x_k)^n \sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1} \frac{x_k}{X}}{W_i} \\
&= \frac{\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mi} c_{nj} h_{m+n} + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} m c_{mi} c_{nj} h_{m+n-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} . \tag{A2}
\end{aligned}$$

The *mean compensated price elasticity* ( $\eta_{lij}$ ) is then derived by adding (A2) to (A1).

(iii) Using equations (3) and (4), the *mean expenditure elasticity* is

$$\begin{aligned}
\varepsilon_{li} &= \sum_{k=1}^K \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i} \\
&= \sum_{k=1}^K \left[ \left( 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1}}{w_{ik}} \right) \frac{w_{ik}}{W_i} \frac{x_k}{X} \right] \\
&= 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} \sum_{k=1}^K \left( \frac{x_k}{X} \right) (\ln x_k)^{m-1}}{W_i} \\
&= 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} \tag{A3}
\end{aligned}$$

Step 2:

The *full price elasticity evaluated at mean income*  $\phi_{2ij}$  and the *expenditure elasticity evaluated at mean income*  $\varepsilon_{2i}$  can be derived directly from equation (2). The *compensated price elasticity evaluated at mean income*  $\eta_{2ij}$  then follows from equation (4).

Step 3:

The biases follow directly from computing  $\phi_{2ij} - \phi_{1ij}$ ,  $\eta_{2ij} - \eta_{1ij}$ , and  $\varepsilon_{2i} - \varepsilon_{1i}$ .

### Proof of Theorem 2:

Step 1:

(i) Using equations (9) and (4), and taking  $\frac{\partial \hat{c}_{0i}(p)}{\partial \ln p_j}$  to be the same as

$\frac{\partial c_{0i}(p)}{\partial \ln p_j} - \hat{c}_{1i} \sum_{n=1}^{\infty} c_{nj} h_n$  (the conventional assumption in working with a model based on aggregate data), the *full price elasticity* is

$$\begin{aligned}
 \phi_{3ij} &= \frac{\partial \ln Q_i}{\partial \ln p_j} \\
 &= -\delta_{ij} + \frac{\frac{\partial \hat{c}_{0i}}{\partial \ln p_j}}{W_i} \\
 &= -\delta_{ij} + \frac{\frac{\partial c_{0i}(p)}{\partial \ln p_j} - \hat{c}_{1i} \sum_{n=1}^{\infty} c_{nj} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m} \\
 &= -\delta_{ij} + \frac{\frac{\partial c_{0i}(p)}{\partial \ln p_j} - \sum_{m=1}^{\infty} m c_{mi} h_{m-1} \sum_{n=1}^{\infty} c_{nj} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m}, \tag{A4}
 \end{aligned}$$

(ii) The second component of equation (7), again using equations (6) and (3), can be written as



$$\begin{aligned}
W_j \frac{\partial \ln Q_i}{\partial \ln X} &= \frac{W_j}{W_i} \left( W_i + \sum_{m=1}^{\infty} m c_{mi} h_{m-1} \right) \\
&= \frac{\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nj} c_{mi} h_n h_m + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} m c_{nj} c_{mi} h_n h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m}
\end{aligned} \tag{A5}$$

The *compensated price elasticity* ( $\eta_{3ij}$ ) is then obtained by adding (A5) to (A4).

(iii) The *expenditure elasticity evaluated at its mean* is

$$\begin{aligned}
\varepsilon_{3i} &= \frac{\partial \ln Q_i}{\partial \ln X} \\
&= 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m}
\end{aligned} \tag{A6}.$$

Step 2:

The biases follow directly from computing  $\phi_{3ij} - \phi_{1ij}$ ,  $\eta_{3ij} - \eta_{1ij}$ , and  $\varepsilon_{3i} - \varepsilon_{1i}$ .

## Appendix B – Biases for Familiar Demand Systems

TABLE B1: BIASES IN USING EM TO ESTIMATE ME  
WITH SELECTED MODELS OF RANK 2, 3 OR 4

FULL PRICE ELASTICITY BIASES ( $\phi_{2ij} - \phi_{1ij}$ )

TLOG:

$$\left[ -\gamma_{ij}^* \sum_k \gamma_{ik}^* h_1 \right] \left[ \alpha_i^* \left( \alpha_i^* - \sum_k \gamma_{ik}^* h_1 \right) \right]^{-1}$$

AIDS:

$$\left[ (\gamma_{ij} - \alpha_j \beta_i) \beta_i h_1 \right] [\alpha_i (\alpha_i + \beta_i h_1)]^{-1}$$

QUAIDS:

$$\left[ (\gamma_{ij} - \alpha_j \beta_i) (\alpha_i + \theta_i) - (\gamma_{ij} - \alpha_j (\beta_i + 2\lambda_i h_1) - \lambda_i \beta_j h_2) \alpha_i \right] [\alpha_i (\alpha_i + \theta_i)]^{-1}$$

L4:

$$\frac{\chi_{0ij} \sum_{m=0}^{\infty} c_{mi} h_m - \left( \sum_{m=0}^{\infty} \chi_{mij} h_m \right) c_{0i}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad \text{with } c_{mi} \text{ defined by equation (14)}$$

COMPENSATED PRICE ELASTICITY BIASES ( $\eta_{2ij} - \eta_{1ij}$ )

TLOG:

$$\left\{ \left[ -\gamma_{ij}^* - \alpha_i^* \sum_k \gamma_{kj}^* - \alpha_j^* \sum_k \gamma_{ik}^* \right] \left[ \sum_k \gamma_{ik}^* h_1 \right] \right. \\ \left. + \left[ \alpha_i^* \left( (-\alpha_i^* \sum_k \gamma_{kj}^* + 2 \sum_k \gamma_{ik}^* \sum_k \gamma_{kj}^*) h_1 + \sum_k \gamma_{ik}^* \sum_k \gamma_{kj}^* h_2 \right) \right] \right\} \left[ \alpha_i^* \left( \alpha_i^* - \sum_k \gamma_{ik}^* h_1 \right) \right]^{-1}$$

AIDS:

$$\left[ (\gamma_{ij} + \alpha_i \alpha_j) \beta_i h_1 - (\alpha_j \beta_i + \alpha_i \beta_j + \beta_i \beta_j) h_1 + \beta_i \beta_j h_2 \right] \alpha_i [\alpha_i (\alpha_i + \beta_i h_1)]^{-1}$$

QUAIDS:

$$\left[ (\gamma_{ij} + \alpha_i \alpha_j) \theta_i - \alpha_i (\alpha_j \beta_i + \alpha_i \beta_j + \beta_i \beta_j) h_1 \right. \\ \left. - \alpha_i ((\alpha_j \lambda_i + \beta_j \beta_i + \lambda_j \beta_i + \beta_j \lambda_i + \alpha_i \lambda_j) h_2 + (\beta_j \lambda_i + \beta_i \lambda_j + 2\lambda_j \lambda_i) h_3 + \lambda_i \lambda_j h_4) \right] [\alpha_i (\alpha_i + \theta_i)]^{-1}$$

L4:

$$\left\{ \left( (\chi_{0ij} + c_{0j} c_{0i}) + c_{0j} c_{1i} \right) \sum_{m=0}^{\infty} c_{mi} h_m - \left( \sum_{m=0}^{\infty} \chi_{mij} h_m + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} m c_{mi} c_{nj} h_{m+n-1} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mi} c_{nj} h_{m+n} \right) c_{0i} \right\} \left( c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m \right)^{-1}$$

with  $c_{mi}, c_{nj}$  defined by equation (14)

# EXPENDITURE ELASTICITY BIASES ( $\varepsilon_{2i} - \varepsilon_{1i}$ )

TLOG:

$$\left( \sum_k \gamma_{ik}^* \right)^2 h_1 \left[ \alpha_i^* \left( \alpha_i^* - \sum_k \gamma_{ik}^* h_1 \right) \right]^{-1}$$

AIDS:

$$\beta_i^2 h_1 [\alpha_i (\alpha_i + \beta_i h_1)]^{-1}$$

QUAIDS:

$$[\beta_i (\beta_i h_1 + \lambda_i h_2) - 2\alpha_i \lambda_i h_1] [\alpha_i (\alpha_i + \theta_i)]^{-1}$$

L4:

$$\frac{c_{1i} \sum_{m=0}^{\infty} c_{mi} h_m - c_{0i} \sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad \text{with } c_{mi} \text{ defined by equation (14)}$$

---

EM – elasticity at mean income; ME – mean elasticity;  $\theta_i = \beta_i h_1 + \lambda_i h_2$ ; the first three values of  $\chi_{im}$  are

$$\begin{aligned} \chi_{0ij} &= \rho_0 \tau_j (\tau_i - \alpha_i) + (1 - \rho_0) \left( \gamma_{ij} + \beta_i \frac{(\rho_0 (\alpha_j - \tau_j) - \alpha_j)}{(1 - \rho_0)} \right); \\ \chi_{1ij} &= (\alpha_i - \tau_j) \tau_j \rho_0 + \rho_0 \left( \gamma_{ij} + \beta_i \frac{(\rho_0 (\alpha_j - \tau_j) - \alpha_j)}{(1 - \rho_0)} \right) + 2\lambda_i \frac{\rho_0 (\alpha_j - \tau_j) - \alpha_j}{(1 - \rho_0)}; \\ \chi_{2ij} &= \frac{1}{2} \left[ \tau_j \rho_0 (\tau_i - \alpha_i) - \rho_0 \left( \gamma_{ij} + \beta_i \frac{(\rho_0 (\alpha_j - \tau_j) - \alpha_j)}{(1 - \rho_0)} \right) - 2\lambda_i \left( \frac{\beta_j}{(1 - \rho_0)} - \frac{\tau_j \rho_0}{(1 - \rho_0)^2} \right) + \right. \\ &\quad \left. \frac{\rho_0 \tau_j \beta_i}{(1 - \rho_0)} + \rho_0 \left( \frac{\rho_0 \tau_j \beta_i}{(1 - \rho_0)^2} - 2\lambda_i \frac{(\tau_j \rho_0 + \alpha_j)}{(1 - \rho_0)} \right) \right]. \end{aligned}$$

TABLE B2: BIASES IN USING AM TO ESTIMATE ME

FULL PRICE ELASTICITY BIASES ( $\phi_{3ij} - \phi_{lij}$ )

TLOG:

$$\left[ -\sum_k \gamma_{kj}^* \sum_k \gamma_{ik}^* h_1 \right] \left[ -\alpha_i^* + \sum_k \gamma_{ik}^* h_1 \right]^{-1}$$

AIDS:

$$[-\beta_i \beta_j h_1] [\alpha_i + \beta_i h_1]^{-1}$$

QUAIDS:

$$[\lambda_i \beta_j h_2 - (\beta_i + 2\lambda_i h_1) \theta_j] [\alpha_i + \theta_i]^{-1}$$

L4:

$$\frac{-\sum_{m=1}^{\infty} \kappa_{mij} h_m - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m} \quad \text{with } c_{mi}, c_{nj} \text{ defined by equation (14)}$$

COMPENSATED PRICE ELASTICITY BIASES ( $\eta_{3ij} - \eta_{lij}$ )

TLOG:

$$\left[ \sum_k \gamma_{ik}^* \sum_k \gamma_{kj}^* (h_1^2 - h_1 - h_2) \right] \left[ -\alpha_i^* + \sum_k \gamma_{ik}^* h_1 \right]^{-1}$$

AIDS:

$$\beta_i \beta_j [h_1^2 - h_1 - h_2] [\alpha_i + \beta_i h_1]^{-1}$$

QUAIDS:

$$[\beta_i \beta_j (h_1^2 - h_2 - h_1) + (\beta_i \lambda_j + \lambda_i \beta_j) (h_1 h_2 - h_3 - h_2) + \lambda_i \lambda_j (h_2^2 - h_4 - 2h_3)] [\alpha_i + \theta_i]^{-1}$$

L4:

$$\frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mi} c_{nj} [h_m h_n - h_{m+n}] - \sum_{m=1}^{\infty} \kappa_{mij} h_m - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m+n-1}}{\sum_{m=0}^{\infty} c_{mi} h_m}$$

with  $c_{mi}, c_{nj}$  defined by equation (14)

# EXPENDITURE ELASTICITY BIASES ( $\varepsilon_{3i} - \varepsilon_{1i}$ )

TLOG:

No Bias

AIDS:

No Bias

QUAIDS:

No Bias

L4:

No Bias

---

AM – elasticity at mean income based on aggregate data; ME – mean elasticity;  $\theta_\ell = \beta_\ell h_1 + \lambda_\ell h_2$  for  $\ell = i, j$ ; the first two values of  $\chi_{im}$  are

$$\begin{aligned} \kappa_{1ij} &= (\alpha_i - \tau_j) \tau_j \rho_0 + \rho_0 \left( \gamma_{ij} - \beta_i \frac{\rho_0 \tau_j}{(1 - \rho_0)} \right) - 2\lambda_i \frac{\rho_0 \tau_j}{(1 - \rho_0)}; \\ \kappa_{2ij} &= \frac{1}{2} \left[ \tau_j \rho_0 (\tau_i - \alpha_i) - \rho_0 \left( \gamma_{ij} - \beta_i \frac{\rho_0 \tau_j}{(1 - \rho_0)} \right) - 2\lambda_i \left( \frac{\beta_j}{(1 - \rho_0)} - \frac{\tau_j \rho_0}{(1 - \rho_0)^2} \right) + \right. \\ &\quad \left. \frac{\rho_0 \tau_j \beta_i}{(1 - \rho_0)} + \rho_0 \left( \frac{\rho_0 \tau_j \beta_i}{(1 - \rho_0)^2} - 2\lambda_i \frac{\tau_j \rho_0}{(1 - \rho_0)} \right) \right]. \end{aligned}$$

TABLE B3: BIASES IN USING AM TO ESTIMATE EM  
WITH SELECTED MODELS OF RANK 2, 3 OR 4

FULL PRICE ELASTICITY BIASES ( $\phi_{3ij} - \phi_{2ij}$ )

TLOG:

$$\left[ \left( \gamma_{ij}^* + \alpha_i^* \sum_k \gamma_{ik}^* \right) \sum_k \gamma_{ik}^* h_1 \right] \left[ \alpha_i^* \left( \alpha_i^* - \sum_k \gamma_{ik}^* h_1 \right) \right]^{-1}$$

AIDS:

$$\left[ -\gamma_{ij} + (\alpha_j \beta_i - \alpha_i \beta_j) \beta_i h_1 \right] [\alpha_i (\alpha_i + \beta_i h_1)]^{-1}$$

QUAIDS:

$$\left[ -\gamma_{ij} \theta_i - 2\lambda_i \alpha_i (\alpha_j + \theta_j) + \beta_i (\alpha_j \theta_i - \alpha_i \theta_j) \right] [\alpha_i (\alpha_i + \theta_i)]^{-1}$$

L4:

$$\frac{\chi_{0ij} \left( c_{0i} - \sum_{m=0}^{\infty} c_{mi} h_m \right) - c_{0i} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad \text{with } c_{mi}, c_{nj} \text{ defined by equation (14)}$$

COMPENSATED PRICE ELASTICITY BIASES ( $\eta_{3ij} - \eta_{2ij}$ )

TLOG:

$$\left[ \left( \gamma_{ij}^* + \alpha_j^* \sum_k \gamma_{ik}^* \right) \sum_k \gamma_{ik}^* h_1 \right] \left[ \alpha_i^* \left( \alpha_i^* - \sum_k \gamma_{ik}^* h_1 \right) \right]^{-1} + \sum_k \gamma_{kj}^* h_1$$

AIDS:

$$-\gamma_{ij} \beta_i h_1 [\alpha_i (\alpha_i + \beta_i h_1)]^{-1} + \beta_j h_1$$

QUAIDS:

$$-\gamma_{ij} \theta_i [\alpha_i (\alpha_i + \theta_i)]^{-1} + \theta_j$$

L4:

$$\left\{ c_{0i} \cdot \left[ \chi_{0ij} + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nj} c_{mi} h_n h_m + \sum_{m=1}^{\infty} m c_{0j} c_{mi} h_{m-1} \right] - [\chi_{0ij} + c_{0j} c_{1i} + c_{0j} c_{0i}] \cdot \sum_{m=0}^{\infty} c_{mi} h_m \right\} \left( c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m \right)^{-1}$$

with  $c_{mi}, c_{nj}$  defined by equation (14)

# EXPENDITURE ELASTICITY BIASES ( $\varepsilon_{3i} - \varepsilon_{2i}$ )

TLOG:

$$-\left(\sum_k \gamma_{ik}^*\right)^2 h_1 \left[ \alpha_i^* \left( \alpha_i^* - \sum_k \gamma_{ik}^* h_1 \right) \right]^{-1}$$

AIDS:

$$-\beta_i^2 h_1 [\alpha_i (\alpha_i + \beta_i h_1)]^{-1}$$

QUAIDS:

$$(2\alpha_i \lambda_i h_1 - \beta_i \theta_i) [\alpha_i (\alpha_i + \theta_i)]^{-1}$$

L4:

$$\frac{c_{0i} \sum_{m=1}^{\infty} m c_{mi} h_{m-1} - c_{1i} \sum_{m=0}^{\infty} c_{mi} h_m}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad \text{with } c_{mi} \text{ defined by equation (14)}$$

---

AM – elasticity at mean income based on aggregate data; EM – elasticity at the mean income;  $\theta_\ell = \beta_\ell h_1 + \lambda_\ell h_2$

$$\text{for } \ell = i, j; \quad \chi_{0ij} = \rho_0 \tau_j (\tau_i - \alpha_i) + (1 - \rho_0) \left( \gamma_{ij} + \beta_i \frac{(\rho_0 (\alpha_j - \tau_j) - \alpha_j)}{(1 - \rho_0)} \right)$$

## APPENDIX C: MICRO PARAMETER VALUES USED IN CALIBRATION

TABLE C1: UNDERLYING EM ELASTICITIES AND  
PARAMETERS OF MICRO MODEL

	FOOD	ALCOHOL	FUEL	CLOTHING	TRANSPORT	SERVICES
EM						
$\phi_{ii}$	-0.564	-1.740	-0.517	-0.622	-0.696	-0.724
$\eta_{ii}$	-0.350	-1.580	-0.450	-0.530	-0.480	-0.550
$\varepsilon_i$	0.610	2.290	0.840	0.920	1.200	1.450
TLOG						
$\alpha_i^*$	-0.35	-0.07	-0.08	-0.10	-0.18	-0.12
$\gamma_{ij}^*$	-0.2005	0.0581	-0.0396	-0.0386	-0.0482	-0.0266
$\sum_j \gamma_{ij}^*$	-0.1365	0.0903	-0.0128	-0.008	0.036	0.054
AIDS						
$\alpha_i$	0.35	0.07	0.08	0.10	0.18	0.12
$\gamma_{ii}$	0.105	-0.0455	0.0376	0.037	0.0612	0.0396
$\beta_i$	-0.1365	0.0903	-0.0128	-0.008	0.036	0.054
QUAIDS						
$\alpha_i$	0.35	0.07	0.08	0.10	0.18	0.12
$\gamma_{ii}$	0.105	-0.0455	0.0376	0.037	0.0612	0.0396
$\beta_i$	-0.1365	0.0903	-0.0128	-0.008	0.036	0.054
$\lambda_i$	-0.008	-0.002	0.037	-0.026	0.015	-0.027
L4						
$\rho_0 = .2$						
$\alpha_i$	0.3325	0.0825	0.0475	0.105	0.185	0.1325
$\gamma_{ii}$	0.1297	-0.0577	0.0417	0.0461	0.0764	0.0487
$\beta_i$	-0.119	0.0778	0.0197	-0.013	0.031	0.0415
$\lambda_i$	-0.008	-0.002	0.037	-0.026	0.015	-0.027
$\tau_i$	0.42	0.02	0.21	0.08	0.16	0.07



TABLE C2: INCOME DISTRIBUTION PARAMETERS FOR SEVEN COUNTRIES

	$h_1$	$h_2$	$h_3$	$h_4$
Sweden	.123	.242	.015	.113
Norway	.142	.259	-.023	.168
Israel	.166	.313	.006	.219
Canada	.171	.321	-.007	.263
United Kingdom	.172	.321	-.002	.232
United States	.204	.367	-.016	.345
Germany	.229	.456	.105	.412

NOTE: Calculations are based on after-tax family income quintile shares provided in table 2 of O'Higgins, Schmaus, and Stephenson (1989). See Denton and Mountain (2004) for details of the calculations. Values for Sweden and Germany are used in calibration for simulation purposes.

### References

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