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ABSTRACT

Population aging is a problem common to many countries: an increasing proportion of retired people, a decreasing proportion of working age, and resultant downward pressure on national product per capita. We explore longer-run aspects of immigration as a policy instrument in this context. We consider, by simulation, the importance of immigrant age distribution, proportion of child immigrants, productivity growth as an offset to aging, possible higher fertility, increased life expectancy, and greater labour force participation among older people. Our laboratory for exploration is a mythical country Alpha with a simple economy and realistic characteristics of an aging population.

1. INTRODUCTION

Immigration is most often viewed by the host country from the point of view of its effects on the economy. Public policy is typically guided by concerns about perceived current or imminent labour shortages, either general or specific to particular skill requirements, and the resultant contribution of immigrants to economic growth is a prominent consideration. However immigrants contribute not only to the supply of labour and aggregate output but to the population as well, and hence to the number of people consuming the output. Aggregate growth of output or income is not itself an indicator of the standard of living in a country. (Popular discussion of the role of immigration might lead one to think otherwise.) Moreover, immigration totals may include dependent children and others not destined for the labour force – consumers who are not also producers. The addition of immigrants changes the future dynamics of a population, and not only its growth rate; the age structure changes also. Immigrant workers eventually retire and become dependents; the children of today's immigrants come of working age, enter the labour force, and bear their own children; and so on. In the longer run there is much more to the story of how immigration affects the economy than just its shorter-run effects on aggregate GDP. Those longer-run effects are the subject of this paper.

The dominant inducement for immigration policy today in many developed countries is population aging – a shift in age structure towards older ages brought about, during the second half of the 20th century, by a sequence of high fertility rates followed by declining and then persistently low rates (see Bongaarts, 1999, on the history of fertility rates). This is the sequence that has resulted today in the prospect of a large proportionate increase in the retired population and a concomitant decrease in the labour force proportion. The prospect of population aging is widespread; it has been recognized by demographers, economists, and others for many years, and its implications studied (see, for example, Masson and Tryon, 1990, Anderson and Hussey, 2000, Denton and Spencer, 2000, Mamolo and

Scherbov, 2009, Vincent and Volkoff, 2010, Lee and Mason, 2011, United Nations, 2013, Scherbov, Sanderson, and Mamolo, 2014, and other references therein). Prominent among developed countries affected by the aging phenomenon are Australia, Canada, France, Germany, Japan, New Zealand, the United Kingdom, and the United States (Anderson and Hussey, 2000).The effects will come sooner and be more pronounced for some countries, later and less pronounced for others, but the changes in age structure and demographic outlook are similar in the main, if not in the details and timing.

We construct a model of a country with an aging population and a government that has available to it immigration as an instrument to offset the consequent effects on the economy – or rather, two instruments: the level of immigration, or quota, and the age distribution of immigrants. The country, which we shall call Alpha, is fictitious and generic, rather than representative of any actual country. We model its economy in a simple fashion, imposing the restriction that immigrants and nonimmigrants are the same with respect to labour force characteristics; only their numbers and ages matter. In this manner we strip away other characteristics (differences in education or skill levels, for example) that may be important in a different context so as to focus exclusively on longer-run demographic dynamics. We calibrate the model and use it in a series of simulation experiments. (For convenience we use Canadian data to calibrate the initial state of the demographic components of the model but the model is not intended to represent Canada.) We consider, in the experiments, alternative immigration strategies and related issues of productivity growth rates, fertility rates, mortality reductions, and labour force participation rates of the older population.

2. THE SETTING

The mythical country of Alpha is our laboratory for studying the longer-run effects of immigration. For simplicity, the population of Alpha is divided into five broad age groups, corresponding to intervals of 20 years: Children (0-19), Young Adults (20-39), Middle Aged (40-59), Seniors (60-79), and

Aged (80-99); there are no survivors beyond 99. It is convenient to refer to each age group and each corresponding time interval as a *generation*. All Children are born to the generation of Young Adult women; the fertility rate for that group is thus identical to the Alphan total fertility rate. Labour force participation is confined to the Young Adult, Middle Aged, and (in much lesser degree) Senior age groups; Children and the Aged have no participation.

Time in Alpha is measured in generations indexed by t, with value 0 at the time of our visit to that country. The population at t = 0 has an important characteristic, a “bulge” in the age distribution inherited from an earlier period of very high fertility – a “baby boom”, as the people of Alpha like to call it. The “baby boom” occurred roughly two generations earlier (at t = -2) and was followed by a “baby bust” – a sharp reduction in fertility and a subsequently maintained low level. The Children of the boom are in Middle Age at t = 0, and a generation later they will be Seniors. “The population is aging,” Alphans would tell us, “and we are greatly concerned about the effects on our economy.” They realize that what a generation ago was a major increase in the size of the labour force will become a major decrease a generation hence, with a sudden increase in the number of elderly dependents. “How will we be able to provide the health care and other services that will be required when the proportion of dependents is increasing and the proportion of workers decreasing? ” the Alphans would ask.

The economy of Alpha is closed to trade and foreign asset ownership (in either direction) but open to immigration – indeed, there is an infinite supply of potential immigrants available, and thus the possibility of using immigration as a policy tool to offset what is going on in the domestic population. (Note that we are talking about immigrants as permanent additions to the population, not temporary “guest workers”.)The government has two instruments in that regard: it can set the immigration quota – the number of immigrants to be admitted in each generation – and (importantly) it can set the age distribution. What follows in this paper is a model and assessment of the long-run effects of using those instruments

3. THE MODEL

The dynamics of the population and the economy of Alpha are simple, and can be modelled accordingly. Let the column vector n stand for the population by age and sex: the first five rows are female age groups (youngest to oldest), the second five are male age groups. The progress of the population from generation t to generation t+1 can be represented as

$$n_{t+1} = Qn_t + m_{t+1} \qquad (1)$$

where m is a vector of immigrants (with age-sex elements corresponding to those of n, and all nonnegative) and Q is a 10x10 Leslie (or projection) matrix. The Leslie matrix is a device well known to demographers and population biologists (Leslie, 1945, 1948); its nonzero elements are determined by age-sex-specific survival rates, the fertility rate, and the male/female birth ratio. If there were no immigration, and all rates were constant, $n_{t+1} = Qn_t$ would hold exactly for all t. (The matrix is defined more precisely in the Appendix.)

There is no emigration in Alpha, only immigration. Alternatively, one could allow emigration as long as emigrants were distributed in the same age-sex proportions as immigrants; m would then be defined as *net* immigration. However, it is more convenient and straightforward to simply rule out emigration.

The vector m can be separated into two components, one representing the total number of immigrants, the scalar M, the other their proportionate age-sex distribution, the vector $\alpha \in A$, where A is the (infinite) set of all possible age-sex distributions. M and α are both set by the government as a matter of policy. (Keep in mind that there is an infinite supply of potential immigrants.)

We refer to M as the *immigration quota*. The quota In Alpha is set as a proportion q of what the total population would be in any given generation *without* immigration. The actual total population in generation t+1 is $u'n_{t+1}$, where u is a column vector of ones, and the total population as it would be if there were no immigration is $u'Qn_t$. The immigration quota is then $M_{t+1} = q(u'Qn_t)$. Making the substitutions, equation (1) can be rewritten as

$$n_{t+1} = Qn_t + M_{t+1}\alpha = Qn_t + q(u'Qn_t)\alpha \qquad (2)$$

The instruments q and α are thus the two available to the government in implementing an immigration policy to offset the effects of population aging on the economy. (There are of course other immigration policy instruments – choice of skill composition, education level, geographic settlement area, etc. – but these are the ones with the greatest and most direct long-run implications for the economy and the population as a whole, and the ones of interest in this paper.)

The employed labour force – or simply labour force, as we shall call it -- is determined by the population vector n and a vector of constant participation rates r, shared by both immigrants and the domestic population: thus $L = r'n$. The rate of unemployment in Alpha is constant and the same for all age-sex groups, and for both domestic and immigrant workers; its effects can be ignored. All workers, domestic and immigrant, are homogeneous in other respects too – education, skill level, hours worked, productivity; only the relative size of the labour force matters to the economy.

Output Z (in real terms) is generated in Alpha by a constant-returns-to-scale Cobb-Douglas production function, with inputs L for labour and K for capital: in log form,

$$\ln Z_t = \mu + \theta t + \beta \ln K_t + (1-\beta)\ln L_t \qquad (3)$$

where θ is the intergenerational rate of neutral technical progress, or equivalently, total factor productivity. Investment I is supported by a constant saving rate γ: thus $I = S = \gamma Z$. The stock of capital is subject to a rectangular or "one horse shay" depreciation function (Hulten and Wykoff, 1981). A unit of stock is undepreciated for one generation, and is then terminated; hence $K = I = \gamma Z$, a convenient simplification for our purposes. Note that since a generation is 20 years, the rectangular depreciation function provides the same number of capital service years, namely 20, as a geometric function with an annual depreciation rate of 5 percent would provide over its infinite lifetime (1/.05 = 20). Substituting γZ for K in equation (3) and rearranging terms allows us to rewrite the Alpha production function in the simpler form

$$\ln Z_t = \varphi + gt + \ln L_t \qquad (4)$$

where $\varphi = (\mu+\beta\ln\gamma)/(1-\beta)$ and $g = \theta/(1-\beta)$. Output Z is now seen to be proportional to labour input, and hence directly responsive to changes in the population that determine the size of the labour force. The productivity growth rate g is interpreted as a labour productivity growth rate that captures the overall effect of changes in total factor productivity.

In national accounting parlance, Z can be regarded as gross domestic product, or equivalently as gross national product, since the economy of Alpha is closed in all respects except immigration. We can define $Y = (1-\gamma)Z$ as net national income (note that generational capital depreciation is γZ) or as consumption. But again the choice of a definition does not matter for purposes of presentation and analysis: the relevant simulation results are shown in index form, and the indexes are identical, whichever definition one chooses. What matters for our purposes is the effect of immigration on the overall level of economic activity in relation to the population and (to put it loosely) the implied level of welfare. We shall refer to the indexes presented in the tables below as *national income indexes.*

The simplest practical measure of economic welfare or well-being for our purposes is national income per capita, Z/N. Age distribution is ignored in this measure – the denominator is an unweighted sum over all age groups. As an experimental alternative (or supplement) we offer also a weighted measure in the tables, Z/N_w ; children are given half-weight in the calculation of N_w in this measure to capture the idea that they consume a smaller share of income than adults (at least that is so in Alpha). Various other measures can be constructed (we have examined several) but the overall interpretation of results would be little affected.

4. SOME GENERAL CONSIDERATIONS

We calibrate the model in the next section and run a series of simulations in the ones following, resulting in a set of tables that explore the effects of immigration and related issues. First though there are some general considerations that may be helpful in thinking about the application of the model and the simulation results.

The age distribution of the population is of first-order importance for the economy. The problem in prospect for the people of Alpha is the result of a distortion of the distribution brought about by the boom/bust sequence of fertility rates in previous generations, and the consequent imminent decline in the proportion of people of working age. The aim of Alphan immigration policy is then to shift the distribution in a different direction by increasing the proportion of working age and decreasing the proportion in the dependency age groups. Obviously that will not be accomplished if the distribution of immigrants is the same as the domestic distribution in every generation. So the focus will be on bringing in working age adults. But there is more to the story.

There are two groups of prime working age in Alpha: Young Adults and the Middle Aged. (Seniors contribute to the labour force also but in only minor degree.) Middle Aged immigrants contribute to the labour force for one generation but then move into the (mainly) dependent Seniors group in the next, and the Aged group in the one after that. Young Adults have the policy advantage of working for two generations before moving on, but they also bear children, and thus contribute to both the working population and the dependent population. In fact, children accompanying their parents may themselves represent a considerable proportion of the immigration quota. (Envisage two situations: in one a mother brings a child with her when she crosses the border – the child is classified as an immigrant; in the other the mother is pregnant when she crosses the border and bears the child shortly after – the child is classified as a member of the domestic population and does not enter into the immigration quota, but the effect is the same.) To go one step further, the children of immigrants are dependents initially but a generation later they are in the labour force, and bearing their own children; and so it goes.

If one thinks about only the first generation effects of immigration, working age at the time of entry is the dominant consideration, even though it brings with it the prospect of more child dependents. If one thinks about effects beyond the first generation, there are other considerations. Immigrants will be part of the population for several generations and will alter the dynamics of the population well beyond the generation in which they enter. One tends to overlook that fact in considering immigration as a means of influencing the economy; one tends rather to focus on shorter-run effects, in a policy context. In the present paper we focus on longer-run effects.

There is also the question of how high to set the quota – how many immigrants to admit in any period. It may be theoretically possible to effect a major shift in population age distribution by setting the quota very high but practical constraints are prohibitive. There are limits to how many newcomers can be absorbed into the society without disruptive effects in any one generation. The question then is how much beneficial effect on the economy can be expected from a realistic quota, given the choice of immigration age distribution. We experiment below with alternative combinations of age distribution and quota size.

Two more points. First, the production function is assumed to be subject to constant returns to scale. If instead there were increasing returns to scale, simply adding immigrants to the labour force would by itself have a positive effect on per capita output, quite aside from other effects that immigration might have, as discussed in subsequent sections of the paper. If there were decreasing returns to scale the addition of immigrants would have some negative effect – again, aside from other effects that we will discuss. Secondly, per capita income, as reported in the tables below, is just a statistical summary measure; by itself it has no distributional implications. But suppose that immigrants were subject to discrimination in the labour market. We have assumed that immigrants and domestic workers are similar in all other respects, in particular that they generate the same marginal products. But if labour market discrimination means that immigrants receive less than their marginal products, and consequently lower per capita incomes, then by implication the domestic population must receive higher per capita incomes. A government considering immigration as a tool to improve the well-being of the population presumably has in mind the benefit to the *domestic* population, at least until, with the passage of time, new immigrants come to be regarded as part of that population. The effect of discrimination would be to transfer to the domestic population some of the product attributable to immigrants. Neither of these effects – scale and discrimination -- is considered again in the analysis; we mention them here just to get them on the record, so to speak.

5. CALIBRATION AND NOTATION

An interesting characteristic of the Alphan population is that it is the same at generation $t = 0$ as the 2001 Canadian census population, and thus exhibits the same distorted age distribution (Statistics Canada, 2013b). Moreover: the Alphan age-sex-specific survival rates incorporated into the Q matrix can be calculated directly from the 2001 Canadian life tables; the initial (total) fertility rate of 1.6 children per woman is the Canadian rate in 2011; and the ratio of male to female births, set at 1.05, is approximately the longstanding Canadian ratio. (See the Appendix for details and references.)

The age-sex labour force participation rates -- ratios of (employed) labour force to population, the elements of the vector r -- are roughly consistent (in broad pattern) with Canadian rates in the decade centered on 2001, with the qualifications that the rates for Children are zero and the rates for Young Adults and Middle Aged are equal. The rates for females, the top half of r, are (0, .75, .75, .10, 0); the rates for males, the bottom half of r, are (0, .85, .85, .20, 0).

Since output Z is proportional to labour input, and results are shown only as indexes, there is no need to set values for φ or the underlying β, γ, μ and θ parameters (equation (4)). The rate of growth of productivity, g, is set to zero in the initial simulations, but allowed to vary in some later ones. (This parameter, or rather the interpretation of its implications, turns out to be very important, as we shall see.)

The simulations involve runs with different immigrant age distributions and some simple notation is helpful in presenting results. First, note that all simulations assume that immigrants in each age group are equally divided between males and females; we do not experiment with differences in sex composition. This cuts to five the number of values that would have to be reported in defining a distribution. Moreover, we assume in most cases (Table 1 is an exception) that immigration policy choices are restricted to Children, Young Adults, and the Middle Aged; no Seniors or Aged immigrants are permitted since immigrants in those age groups would simply add to the numbers of dependents (aside from a small proportion of Seniors who enter the labour force). Our focus is on immigration as a policy instrument for influencing the economy, and offsetting the effects of population aging. Permitting older immigrants to enter might certainly be considered desirable for other reasons but its effect on immigration as an economic policy tool would obviously be to weaken it. A practical result of this exclusion for presentation purposes is that the number needed to be reported in defining an immigration age distribution is now reduced to three. We choose the symbol AGEIM to stand for “age distribution of immigrants” and report the proportions in percentage form. Writing AGEIM (25, 50, 25) in a column heading in a table, for example, means that immigrants are distributed as 25 percent Children, 50 percent Young Adults, and 25 percent Middle Aged .

6. INITIAL SIMULATIONS

We begin, in Table 1, with two sets of initial simulations. The starting population (t = 0) is shown in the first column of figures. The next three show the evolution of the population over three generations, assuming no immigration. The final three introduce immigration and trace the evolution again, assuming three alternative immigration quotas, each coupled with an age distribution identical to that of the initial (t = 0) population.

When there is no immigration the population of Alpha increases by 4.5 percent in the first generation, and then decreases thereafter; in fact, with the fertility rate constant at 1.6 children per woman the population would decline from generation to generation indefinitely. (The fertility rate required for the population to stabilize, in both total and age distribution, in the long run – the “natural replacement rate” -- is approximately 2.07. We experiment with different rates in later simulations.) The proportion of old people (Seniors plus Aged) increases from 16.8 percent at t = 0 to 26.6 percent at t = 1, and then almost doubles the initial level, rising to 31.5 and 31.9 percent. Concomitantly, the proportion of Children decreases. The ratio of labour force to population falls from 48.1 percent at t = 0 to 44.5 percent at t = 1, and then to 42.2 and 41.7 percent, producing sharp declines in the national income index: from a base of 100.0 at t = 0, income falls to 96.8 at t = 1, 86.1 at t = 2, and 75.6 at t = 3. Income per capita falls accordingly, but less precipitously after one generation, since the population is also declining: the unweighted measure falls to 92.6, 87.9, and 86.8; the weighted measure falls even more – to 90.7, 85.2, and 84.2. Such is the population/economy trajectory in store for Alphans in the absence of immigration. We have run the simulations out for several more generations beyond the three for which results are shown in the table but the longer -run pattern is clear after three: a continuing high proportion of old people relative to the base generation, a continuing lower proportion of children, a much reduced labour force-to-population ratio, a declining national income, and a much lower level of income per capita, weighted or unweighted.

Immigration is introduced in Table 1 (and in subsequent tables) at three quota levels: 10, 20, and 30 percent per generation. (The corresponding annual rates are approximately .48, .92, and 1.32 percent; a sustained level of .48 would be considered rather high by modern international standards for a developed country, and 1.32 as very high.) As noted above, the age distribution chosen for this first set of simulations with immigration is the distribution of the population as it was at t = 0. There is no suggestion that this is an ideal distribution; it is chosen simply as an initial reference case. (A naïve argument by the Alphan authorities in support of the choice would be that the economy was doing well at t = 0, with the age distribution that prevailed at that time, so let us bring in immigrants with the same distribution.) One effect is to stop the decline of the population (with the exception of a very slight dip when q = 10 percent, at t = 3). The proportion of old people is a little lower than in the no-immigration case and the labour force/population ratio a little higher, although it takes a very high quota rate to have much effect in that regard. The immediate decline of national income is arrested: with q = 10 percent income roughly levels off; it increases significantly with q = 20 percent, and rapidly with q = 30 percent. But income per capita (either measure) never recovers; it is higher than the corresponding no-immigration level in all cases but still well below what is was at t = 0. In short, bringing in immigrants with the base level age distribution can moderate the economic decline induced by population aging, but only in limited degree if one takes account of the effect of immigration on the size of the population as well as the level of economic activity, and only with a high quota level. Let us see now what effect altering the age distribution of immigrants might have.

7. IMMIGRATION WITH WORKING AGE CONCENTRATION

Choosing an age distribution with a high concentration of immigrants in the working ages – Young Adults and Middle Aged – makes a big difference. Table 2 assumes two such distributions: (a) 50 percent Young Adults, 25 percent Middle Aged, plus 25 percent Children; (b) 75 percent Young Adults, no Middle Aged, plus 25 percent Children. Both distributions raise the labour force/population ratio and increase the level of national income per capita (either measure) above what it would have been had there been no immigration, and also above the level resulting from the immigrant age distribution assumed in Table 1. The effects are greater, the higher the quota. The immediate effect (t = 1) is the same for both distributions but by the second generation (t = 2) the Middle Aged immigrants admitted previously under distribution (a) have become Seniors, and thus started to add to the dependent population. Under distribution (b) this effect is delayed until the third generation (t = 3).

A fraction of the decline in income per capita from the base period is offset under either distribution. The quota matters greatly in this regard but whatever the quota, the distribution with the higher proportion of Young Adults dominates. However, even that one requires a high quota to eliminate the decline; to come close requires a quota of 20 percent, to eliminate the decline entirely requires a quota of 30 percent, and then not until the second generation.

Note that any of the immigration plans considered in Table 2 and other tables would do away with the prospect of a decreasing population. The rate of population growth is a result of the quota choice, but to some extent also the choice of immigrant age distribution since a higher proportion of Young Adult immigrants means subsequently a larger number of births.

8. THE EFFECT OF ELIMINATING CHILD IMMIGRANTS

Child immigrants augment immediately the dependent component of the population and it is of interest therefore to explore the consequences of restricting admission to adults. We do that in Table 3. The two immigration choices in this table repeat the distributions of adult immigrants in Table 2 but now stipulate no Child immigrants; the quotas remain the same but the immigration totals consist entirely of adults. The effects are immediate and significant. The income per capita indexes are higher than they were with Children included, in all cases, and the decline from base level is eliminated, all but eliminated, or even converted to an increase with quotas of 20 and 30 percent coupled with the most highly concentrated of the two adult age distributions. Exact results depend on whether one uses the weighted or unweighted per capita measure for comparison but the general nature of the effects is clear: excluding Child immigrants raises per capita national income above what it would otherwise have been, both immediately and in subsequent generations.

9. THE EFFECTS OF IMMIGRATION ON NATIONAL INCOME: A COMPARATIVE VISUAL SUMMARY

We bring together now, in two figures, the simulated results relating to the effects on national income of the fifteen alternative immigration strategies considered in Tables 1 – 3. Figure 1 shows results for (a) aggregate income and (b) population; Figure 2 shows results for (c) unweighted income per capita and (d) weighted income per capita. In the tables the results are shown in the form of indexes, with base 100 representing initial ($t = 0$) levels; in the figures they are shown as percentage differences from initial levels.

In the absence of immigration, national income decreases in each generation ($t = 1, 2, 3$), as shown in Figure 1. With the introduction of immigration, it *increases* in each generation, for all three quota levels and all five age distributions. But the population increases also: immigration means a larger national product but also a larger population and a larger number of consumers to draw income and share the product. So now the differences in rates of growth of national income and population become important, and it is apparent, in Figure 2, that national income *per capita* is sensitive to both the immigration quota and the way in which immigrants are distributed by age.

There are several points to note with regard to Figure 2. The first is the sharp decline in per capita income (unweighted or weighted) when there is no immigration (category A in the figure). The second is the weakness of the offset provided by immigration when immigrants are distributed by age as in the initial Alpha population (category B). Following on from that, the third is the importance of a concentration of immigrants in the working ages, Young Adults and Middle Aged (see categories C and D). A fourth is the greater effect of immigration when Child immigrants are excluded (categories E and F). A fifth (to no surprise) is that the effects increase with the size of the immigration quota. (A quota of 30 percent can virtually eliminate decreases in per capita income or – with a highly concentrated age distribution – convert the decreases into increases.) A sixth is the difference between the effects on unweighted and weighted per capita income: the unweighted measure is somewhat more responsive to immigration than the weighted one (which assigns a lower weight to children), although the directions of effect are the same for both

10. THE IMPLICATIONS OF QUOTA/AGE DISTRIBUTION CHOICES: A CLOSER LOOK

The choice of a quota establishes the total number of immigrants in any generation as a ratio to the population, calculated as it would be if there were no immigrants. We experiment with three quotas, 10, 20, and 30 percent. Policy makers can judge whether these quotas are acceptable in relation to the overall size of the population or whether they pose difficulties in absorbing the resulting numbers of new immigrants into the Alphan society. But the choice of an age distribution takes the absorption issue further; it invites the question of whether the implied number of immigrants *in each age group* is acceptable. We consider now, from that point of view, the number of immigrants as a proportion of the population in each group. We do this for generation 1 and show the results in Table 4.

Referring back to section 3, the total population in generation 1 can be obtained from equation (2) as

$$N_1 = u'n_1 = u'n_0 + q(u'Qn_0) \qquad (5)$$

where u is again a vector of ones. Let b_1 be the vector of age-sex proportions of the overall population in generation 1 (corresponding to α, the age-sex proportions vector for immigrants). We may then write

$$n_1 = N_1b_1 = (u'Qn_0 + q(u'Qn_0))b_1 \qquad (6)$$

The immigration total is $M_1 = q(u'Qn_0)\alpha$. Letting diag(α) and diag(b_1) be diagonal matrices in which α and b_1 are the diagonals, we write

$$H = (q(u'Qn_0)\mathrm{diag}(\alpha))(u'Qn_0 + q(u'Qn_0)\mathrm{diag}(b_1))^{-1} = (q/(1+q))\mathrm{diag}(\alpha)\mathrm{diag}(b_1)^{-1} \qquad (7)$$

The age-sex-specific immigrant proportions are the diagonal elements of H and the overall share proportion is $M_1/N_1 = q/(1+q)$. Age-specific (male plus female) share proportions based on equation (7) are shown in Table 4 for the three immigration quotas and the alternative age distributions used in the earlier tables.

Age distributions with concentrations in the working age groups can increase markedly the level of national income per capita, as shown in Tables 2 and 3 and Figure 2. But a concomitant of that is a high proportion of immigrants in those particular groups and possible difficulties of absorbing the implied large numbers of newcomers of a given age into the society. The issue of absorption lies outside our model framework but it is something that the government of Alpha would have to consider. The extreme situations in both national income benefits and possible absorption difficulties occur when only

Young Adult immigrants are admitted to the country – the distribution (0, 100, 0). With a quota of 10 percent, 29 percent of the population in that age group are immigrants; with a 20 percent quota, 45 percent are immigrants; and with a 30 percent quota the proportion is well over half, 55 percent. Even with the somewhat less concentrated (0, 67, 33) distribution the proportion in the Young Adult age group reaches 35 percent with a 20 percent quota and 45 percent with a quota of 30 percent. The policy choice that the government of Alpha must make poses a tradeoff – accepting a lower level of income per capita than what might be attainable through immigration vs. possible societal absorption difficulties with a higher immigration quota.

11. PRODUCTIVITY GROWTH AS AN OFFSET TO POPULATION AGING

The rate of growth of productivity is denoted by g in equation (4), section 3. We have set g to zero in all of the simulations thus far. Now we experiment with positive values. The immigration quota and age distribution are instruments under full government control in Alpha; the rate of productivity growth is not. The government may be able to nudge the rate a little by this or that policy but the extent of its influence is no doubt quite limited. Nevertheless, it is of interest to see how productivity growth might act as an offset to the negative effect of population aging on the economy.

Table 5 shows what would happen to national income per capita (unweighted) if a productivity growth rate of 5 or 10 percent were coupled with an immigration quota of 0, 10, 20, or 30 percent, using the (25, 50, 25) age distribution for the calculations in these experiments. (A productivity growth rate of 5 percent per generation is equivalent to an annual rate of .24 percent; a growth rate of 10 percent per generation is equivalent to an annual rate of .48 percent.)

The results in Table 5 appear striking at first glance. Productivity growth of 10 percent per generation by itself, with no immigration, would wipe out immediately (t = 1) the decline of national income per capita brought about by population aging, and raise the income per capita level further in subsequent generations. Coupling even a 5 percent growth rate with positive immigration quotas would set an upward trajectory for income per capita. It would seem then that even a modest rate of productivity growth would eliminate all concerns about the economic effects of population aging. However, that interpretation is superficial.

Suppose, to make a point, that while the productivity growth rate in Alpha is 5 percent, the growth rate in the rest of the world is 10 percent. *Relative to other countries* Alpha's national income per capita would then fall by about 4.5 percent in the first generation (on top of whatever was the decline resulting from population aging); by 8.9 percent in the second; and so on. The point is that to be interpreted realistically, the productivity growth rate should be defined as the difference from the growth rate in the rest of the world, or from the neighbouring country of Beta, perhaps, depending on what is the relevant standard of comparison in Alpha. Moreover, If Alpha had an open rather than a closed economy it would find its terms of trade deteriorating and its relative standard of living falling as a result of its slower productivity growth. If g is defined as a *differential* rate of productivity growth, a positive rate would indeed offset some or all of the effects of population aging on the economy. Zero productivity growth, as we have assumed in the earlier simulations, would then imply that productivity was growing in Alpha at the same rate as elsewhere and that income was measured correspondingly, in relative terms. (Productivity growth itself would of course have no effect on the population; the population would still be on a declining path unless there were an offset provided by immigration or the fertility rate were to increase to the natural replacement level or higher.)

12. WHAT IF THE FERTILITY RATE WERE TO INCREASE?

The "natural replacement" fertility rate is a little under 2.1 children per woman. That is the rate required for the population to achieve a stationary state in the long run – constant population size and an unchanging age distribution. A higher rate would result in continuous population increase, a lower rate in continuous population decline. The rate in Alpha has been 1.6. What if the rate were to increase?

Letting F stand for fertility rate, we experiment with two higher levels, starting at t = 1: the levels are F = 2.0745 (the natural replacement rate to four decimal places) and F = 2.5, a value well above replacement. The particular question of interest is whether such higher rates would add to or diminish the effects of population aging on the economy. The fertility rate, like the productivity growth rate, is not something under the direct control of the government of Alpha, but again the government may be able to influence it to some degree by policies that provide incentives to families to bear more children. Would that be a good idea from the point of view of the national standard of living, as represented by income per capita? It would obviously lower the average age of the population. Would it be a good alternative or supplement to an immigration policy?

The results of the experiments are presented in Table 6. To isolate fertility effects we assume no immigration. The top panel of the table repeats the no-immigration results from Table 1, with the fertility rate held at 1.6. The middle and bottom panels show results for the two higher fertility rates.

With F equal to the replacement rate, the population increases more rapidly at t = 1, and remains at the higher level thereafter, thus arresting the long-run population decline observed previously. But a higher value of F means more children in the first generation, more dependents in the population, a lower labour force/population ratio, and a lower level of national income per capita. The unweighted per capita income index has dropped significantly, from 92.6 (when the fertility rate was 1.6) to 87.0 with the new higher rate; the weighted index has dropped somewhat less, from 90.7 to 87.5. In the second generation (t = 2) the children of the first have come of working age but a new cohort of child dependents has taken their place, and there are only small changes in the labour force/

population ratio and per capita income indexes. There are some further differences in generation 3 but overall the picture is generally similar to that of generation 2.

Much the same can be said, qualitatively, for the results of the further increase in fertility rate to 2.5. What were smaller effects with replacement fertility though have now become bigger ones. Most notably, the reduction of per capita income (weighted or unweighted) in the first generation is much greater.

In sum, the effect on the economy of an increase in the fertility rate in the first generation can be large and unfavourable, from the point of view of income per capita, owing to the addition of more child dependents. The effects in the subsequent generations, when the earlier-generation children have entered the labour force, are smaller, and somewhat mixed. A policy attempt to increase the fertility rate would appear to be an undesirable choice for Alpha from the point of view of its economy, under the assumptions of the model, even if the policy could be effective, which itself is much in doubt. (On the other hand it might well be desirable if moderating or eliminating the prospect of population decline were a goal in itself, without resorting to immigration.)

13. EFFECTS OF REDUCED MORTALITY AND INCREASED PARTICIPATION OF SENIORS

The simulations to this point have assumed constant mortality and labour force participation rates. We experiment now with declining mortality rates (which allow older Alphans to live longer, on average), considered alone and in combination with increased participation of seniors, both with and without concurrent immigration. The mortality assumption is that age-sex death rates would decline over the next three generations at the same average proportionate rates of change as in the last three generations. (By coincidence, these rates of change in Alphan death rates are identical to the corresponding Canadian life table rates of change calculated over the 60-year period 1941 – 2001; see Dominion Bureau of Statistics, 1947, and Statistics Canada, 2006, for the life tables used in the calculations.) The participation assumption is that participation rates of Seniors would increase by half in the first generation, and stay at the new levels in the subsequent two; that means that the participation rate for males would increase from 20 percent to 30 percent, the rate for females from 10 percent to 15 percent. The assumptions about accompanying immigration are a 20 percent quota and a (25, 50, 25) age distribution. The results of the experiments are presented in Table 7. The top panel in the table repeats results from Tables 1 and 2: constant mortality rates are assumed, with and without immigration. The middle panel assumes declining mortality and the bottom one declining mortality plus increased participation rates, with and without immigration in both cases.

The most prominent effects of declining mortality, taken alone, is to increase the proportion of older dependents in the population, decrease the labour force/population ratio, and lower both measures of income per capita. Immigration operates in the opposite direction, and much more strongly, but that is an effect that we have seen before. Introducing increased participation of seniors in the bottom panel of the table offsets the increased dependency effect of lower death rates and has a net positive effect on income per capita, but immigration is again the dominant contributor. In short, declining mortality lowers per capita income, declining mortality plus increasing Seniors' participation rates by half raises it, but while the net effect is significant it takes second place to the effect of immigration. Obviously, other assumptions could be made, and could produce a greater impact; the rates of participation could be increased further, in particular. However, assuming an increase of fifty percent in the rates would already seem to be a rather strong assumption.

14. SUMMING UP

The mythical country of Alpha has been the laboratory for our experiments. Alpha has a simple economy, easily modeled, and demographic characteristics that (conveniently) are the same as those of Canada. Its population is classified according to five broad age groups, or generations, as they are called, and time is measured in generations also, a feature consistent with our focus on long-run change. Our experiments have taken the form of simulations under different assumptions about immigration, productivity growth, fertility increase, mortality decline, and the labour force participation of seniors. The principal aim of our experiments has been to explore the effectiveness of immigration policy in offsetting the economic consequences of population aging. There is an unlimited supply of potential immigrants and the government has two instruments with which to work, the immigration quota in each generation and the immigrant age distribution.

Alpha faces a problem common to many developed countries: a shift in the age distribution of the population towards a lower proportion in the labour force and consequent downward pressure on national income per capita. Immigration can be used to moderate the shift but to be effective the quota level may have to be high, the distribution of adult immigrants highly concentrated in the working ages, and the proportion of child immigrants low. While immigration will bring about an increase in aggregate national income it will also add to the number of consumers sharing in the increase. The worker/dependent ratio among immigrants is therefore a fundamental consideration in policy design. A larger quota will of course produce a larger effect but how large a quota is acceptable from a social point of view is another fundamental consideration. A higher level of productivity could offset the aging-induced decline in per capita income but to be realistically interpreted, productivity would have to be defined in relative terms – relative to the level in the rest of the world, that is. An increase in fertility would raise the proportion of dependents in the population and lower per capita income, both immediately and subsequently. Falling death rates and rising life expectancy would increase the proportion of older dependents; that could be offset by higher labour force participation rates of older people but the increases would have to be proportionately large, and even then might provide only a modest contribution.

15. CONCLUDING REMARK

The people of Alpha join with the authors in hoping that the explorations in this paper provide some helpful guidance in understanding the issues involved in considering immigration policy as a means of dealing with the economic implications of population aging.

APPENDIX: THE LESLIE MATRIX

The Leslie (or projection) matrix Q used in equation (1) and subsequent equations is the 10 x 10 matrix shown in Table A1. The first five rows are for female age groups, youngest to oldest; the next five rows are for males. The entry in the Q(1,2) cell represents the calculation of female children, incorporating an adjustment for newborn mortality: F is the fertility rate (applied to Young Adult females), r_f is the proportion of females at birth, and s_{f0} is the survival rate for female births; the entry in the Q(6,2) cell, $s_{m0}r_mF$, represents the corresponding calculation for male children. The group-to-group survival rates for females are provided in cells Q(2,1), Q(3,2), Q(4,3), Q(5,4); the corresponding rates for males are provided in cells Q(7,6), Q(8,7), Q(9,8), Q(19,9).

The Q matrix can be applied sequentially to project an initial population vector n_0 k generations ahead, ignoring immigration and assuming all rates constant: $n_1 = Qn_0$, $n_2 = Qn_1$, …, $n_k = Qn_{k-1}$ or, more compactly, $n_k = Q^k n_0$. (For discussion of Leslie matrices, their characteristics and application, see Keyfitz and Caswell, 2005, Chapter 7.)

The survival rates in Q are calibrated using 2001 Canadian life tables. (The tables are based on deaths in the years 2000, 2001, 2002 but are commonly referred to as 2001 tables (Statistics Canada, 2006.) F is set initially at 1.6 children per woman, the total fertility rate in Canada in 2011 (Statistics Canada, 2013a). The ratio of males to females at birth is set at 1.05, yielding .488 and .512 as the female and male proportions, approximately the longstanding proportions in Canada. (The ratio 1.05 is within a normal range: "In the absence of manipulation, the sex ratio at birth is remarkably consistent across human populations, with 105 – 107 male births for every 100 female births," Hesketh and Xing, 2006, p. 13271.)

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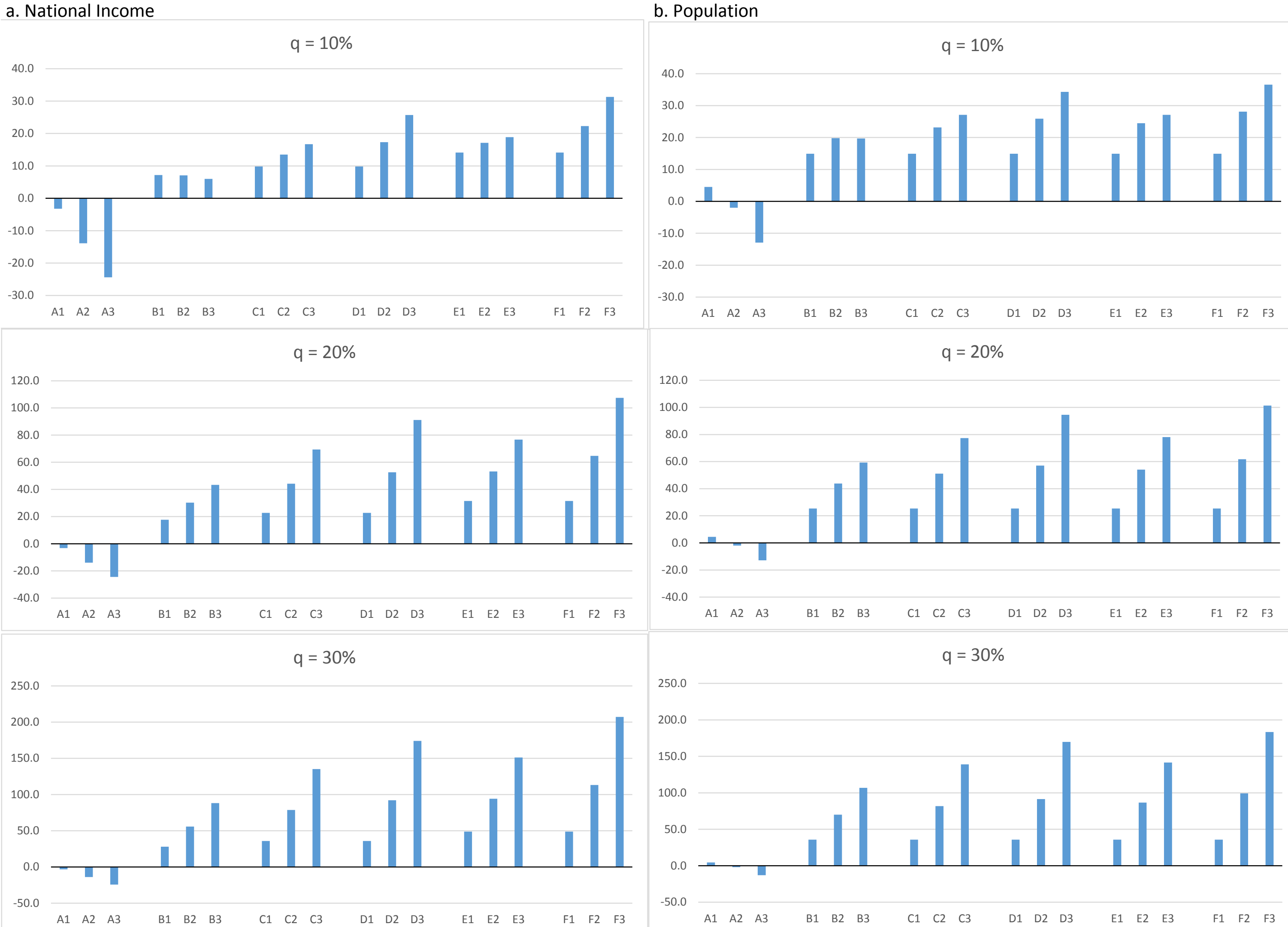
Figure 1: Simulated Changes in National Income and Population for Alpha Under Alternative Immigration Quotas and Age Distributions -- Percent Differences from Initial (t = 0) Level

a. National Income b. Population

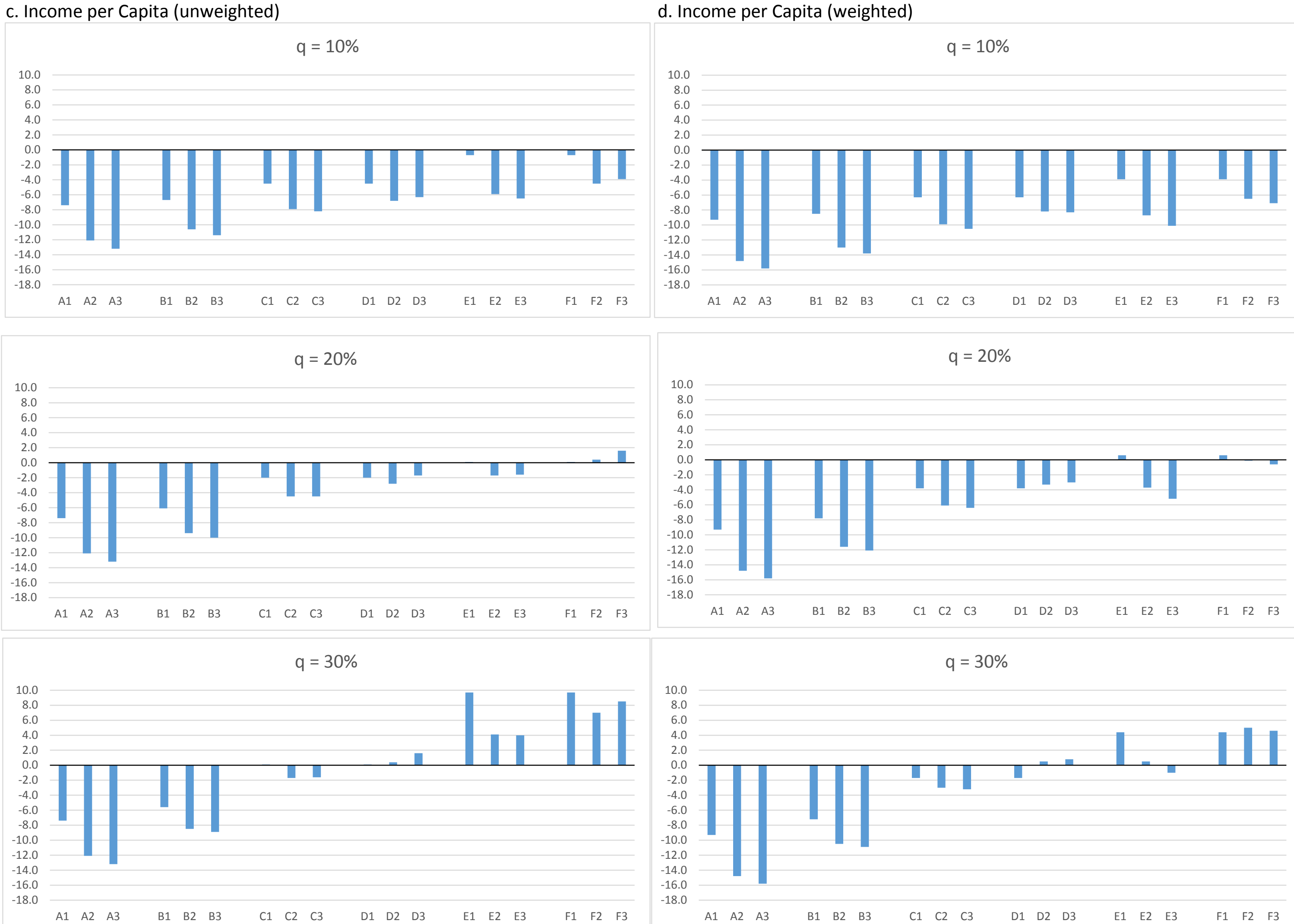


Note: A - no immigration; B - AGEIM(like initial population); C - AGEIM(25,50,25); D - AGEIM(25,75,0); E - AGEIM(0.67,33); F - AGEIM(0,100,0). Generation (t = 1, 2, 3) is indicated by the number attached to an immigration code.

Figure 2: Simulated Changes in Income per Capita for Alpha Under Alternative Immigration Quotas and Age Distributions -- Percent Differences from Initial (t = 0) Level

c. Income per Capita (unweighted)

d. Income per Capita (weighted)



Note: A - no immigration; B - AGEIM(like initial population); C - AGEIM(25,50,25); D - AGEIM(25,75,0); E - AGEIM(0.67,33); F - AGEIM(0,100,0). Generation (t = 1, 2, 3) is indicated by the number attached to an immigration code.

Table 1. Simulations for Alpha with and without Immigration; Immigrants Distributed by Age as in the Initial Population

		No immigration			AGEIM like initial population		
	t = 0	t = 1	t = 2	t = 3	t = 1	t = 2	t = 3
					------------- q = 10% -----------------		
Population	100.0	104.5	98.0	87.1	114.9	119.8	119.7
- growth rate	--	4.5	-6.2	-11.1	14.9	4.3	-0.1
- proportion old	16.8	26.6	31.5	31.9	25.7	29.7	30.0
- proportion child	25.7	21.9	20.2	20.3	22.3	20.9	21.0
LF/Pop. ratio	48.1	44.5	42.2	41.7	44.8	43.0	42.6
National income	100.0	96.8	86.1	75.6	107.2	107.1	106.0
- per capita	100.0	92.6	87.9	86.8	93.3	89.4	88.6
- wtd. per capita	100.0	90.7	85.2	84.2	91.5	87.0	86.2
					------------- q = 20% -----------------		
Population	100.0	104.5	98.0	87.1	125.4	143.8	159.3
- growth rate	--	4.5	-6.2	-11.1	25.4	14.7	10.8
- proportion old	16.8	26.6	31.5	31.9	25.0	28.3	28.5
- proportion child	25.7	21.9	20.2	20.3	22.5	21.4	21.5
LF/Pop. ratio	48.1	44.5	42.2	41.7	45.1	43.5	43.2
National income	100.0	96.8	86.1	75.6	117.7	130.3	143.4
- per capita	100.0	92.6	87.9	86.8	93.9	90.6	90.0
- wtd. per capita	100.0	90.7	85.2	84.2	92.2	88.4	87.9
					------------- q = 30% -----------------		
Population	100.0	104.5	98.0	87.1	135.8	170.0	206.8
- growth rate	--	4.5	-6.2	-11.1	35.8	25.2	21.6
-proportion old	16.8	26.6	31.5	31.9	24.3	27.1	27.3
- proportion child	25.7	21.9	20.2	20.3	22.8	21.8	21.9
LF/Pop. ratio	48.1	44.5	42.2	41.7	45.4	44.0	43.8
National income	100.0	96.8	86.1	75.6	128.1	155.7	188.3
- per capita	100.0	92.6	87.9	86.8	94.4	91.5	91.1
- wtd. per capita	100.0	90.7	85.2	84.2	92.8	89.5	89.1

Note: Population and income variables are indexes; all other variables are percentages. Proportion old is percentage of Seniors and Aged combined; wtd. per capita income assigns half weights to children. The initial population age distribution is (25.7, 29.2, 28.3, 13.8, 3.0).

Table 2. Simulations for Alpha with Alternative Immigrant Age Distributions When There are Child Immigrants

		AGEIM (25, 50, 25)			AGEIM (25, 75, 0)		
	t = 0	t = 1	t = 2	t = 3	t = 1	t = 2	t = 3
		------------------------------------ q = 10% --------------------------------					
Population	100.0	114.9	123.2	127.1	114.9	125.9	134.3
- growth rate	--	14.9	7.2	3.2	14.9	9.5	6.7
- proportion old	16.8	24.2	26.8	27.5	24.2	24.5	25.4
- proportion child	25.7	22.2	21.7	21.3	22.2	22.9	22.0
LF/Pop. ratio	48.1	45.9	44.3	44.1	45.9	44.8	45.0
National income	100.0	109.8	113.5	116.7	109.8	117.3	125.7
- per capita	100.0	95.5	92.1	91.8	95.5	93.2	93.7
- wtd. per capita	100.0	93.7	90.1	89.5	93.7	91.8	91.7
		------------------------------------ q = 20% --------------------------------					
Population	100.0	125.4	151.1	177.4	125.4	157.0	194.6
- growth rate	--	25.4	20.5	17.4	25.4	25.2	24.0
- proportion old	16.8	22.2	23.3	24.1	22.2	19.6	20.7
- proportion child	25.7	22.4	22.8	22.1	22.4	24.7	23.4
LF/Pop. ratio	48.1	47.1	45.9	45.9	47.1	46.7	47.2
National income	100.0	122.8	144.3	169.4	122.8	152.6	191.2
- per capita	100.0	98.0	95.5	95.5	98.0	97.2	98.3
- wtd. per capita	100.0	96.2	93.9	93.6	96.2	96.7	97.0
		------------------------------------ q = 30% --------------------------------					
Population	100.0	135.8	181.8	238.9	135.8	191.3	269.7
- growth rate	--	35.8	33.9	31.4	35.8	40.9	40.9
-proportion old	16.8	20.5	20.5	21.4	20.5	16.1	17.3
- proportion child	25.7	22.6	23.5	22.7	22.6	25.9	24.4
LF/Pop. ratio	48.1	48.1	47.2	47.3	48.1	48.2	48.8
National income	100.0	135.9	178.7	235.1	135.9	192.1	274.0
- per capita	100.0	100.1	98.3	98.4	100.1	100.4	101.6
- wtd. per capita	100.0	98.3	97.0	96.8	98.3	100.5	100.8

Note: See relevant parts of note to Table 1.

Table 3. Simulations for Alpha with Alternative Immigrant Age Distributions When There are No Child Immigrants

		AGEIM (0, 67, 33)			AGEIM (0, 100, 0)		
	t = 0	t = 1	t = 2	t = 3	t = 1	t = 2	t = 3
		---------------------------------- q = 10%			---------------------------------		
Population	100.0	114.9	124.5	127.1	114.9	128.1	136.6
- growth rate	--	14.9	8.4	2.1	14.9	11.5	6.7
- proportion old	16.8	24.2	27.1	29.4	24.2	24.1	26.5
- proportion child	25.7	19.9	20.4	18.6	19.9	21.9	19.7
LF/Pop. ratio	48.1	47.7	45.2	45.0	47.7	45.9	46.2
National income	100.0	114.1	117.1	118.9	114.1	122.3	131.3
- per capita	100.0	99.3	94.1	93.5	99.3	95.5	96.1
- wtd. per capita	100.0	96.1	91.3	89.9	96.1	93.5	92.9
		---------------------------------- q = 20%			---------------------------------		
Population	100.0	125.4	154.1	178.1	125.4	161.8	201.4
- growth rate	--	25.4	22.9	15.6	25.4	29.1	24.4
- proportion old	16.8	22.2	23.7	26.8	22.2	19.1	22.1
- proportion child	25.7	18.3	20.1	17.6	18.3	22.5	19.4
LF/Pop. ratio	48.1	50.4	47.8	47.7	50.4	48.9	49.5
National income	100.0	131.5	153.2	176.7	131.5	164.7	207.5
- per capita	100.0	104.9	99.4	99.2	104.9	101.8	103.0
- wtd. per capita	100.0	100.6	96.3	94.8	100.6	99.9	99.4
		---------------------------------- q = 30%			---------------------------------		
Population	100.0	135.8	186.6	241.5	135.8	199.3	283.3
- growth rate	--	35.8	37.4	29.4	35.8	46.7	42.2
-proportion old	16.8	20.5	21.1	24.5	20.5	15.5	18.7
- proportion child	25.7	16.9	19.5	16.8	16.9	22.4	19.1
LF/Pop. ratio	48.1	52.7	50.0	50.0	52.7	51.4	52.1
National income	100.0	148.9	194.3	251.2	148.9	213.2	307.3
- per capita	100.0	1O9.7	104.1	104.0	109.7	107.0	108.5
- wtd. per capita	100.0	104.4	100.5	99.0	104.4	105.0	104.6

Note: See relevant parts of note to Table 1.

Table 4. Immigrants in Generation 1 as Percentage of Alpha Population, by Age Group, Based on Alternative Choices of Immigration Quota and Age Distribution

	Immigration age distribution (AGEIM)				
	Like initial pop.	(25, 50, 25)	(25, 75, 0)	(0, 67, 33)	(0, 100, 0)
		---------------------------------- q = 10% ---------------------------------			
Children	10.5	10.2	10.2	0.0	0.0
Young Adults	10.7	17.0	23.5	21.6	29.1
Middle Aged	9.5	8.4	0.0	10.9	0.0
Seniors	5.8	0.0	0.0	0.0	0.0
Aged	6.7	0.0	0.0	0.0	0.0
All ages	9.1	9.1	9.1	9.1	9.1
		---------------------------------- q = 20% ---------------------------------			
Children	19.0	18.6	18.6	0.0	0.0
Young Adults	19.3	29.1	38.1	35.4	45.0
Middle Aged	17.3	15.5	0.0	19.6	0.0
Seniors	11.0	0.0	0.0	0.0	0.0
Aged	12.5	0.0	0.0	0.0	0.0
All ages	16.7	16.7	16.7	16.7	16.7
		---------------------------------- q = 30% ---------------------------------			
Children	26.0	25.5	25.5	0.0	0.0
Young Adults	26.4	38.1	47.9	45.2	55.2
Middle Aged	23.8	21.7	0.0	26.7	0.0
Seniors	15.6	0.0	0.0	0.0	0.0
Aged	17.3	0.0	0.0	0.0	0.0
All ages	23.1	23.1	23.1	23.1	23.1

Table 5. Simulations of National Income per Capita for Alpha Assuming Alternative Rates of Productivity Growth (g), with and without Immigration (q)

	t = 0	t = 1	t = 2	t = 3
q = 0 (no immig.)				
g = 0	100.0	92.6	87.9	86.8
g = 5%	100.0	97.2	96.9	100.5
g = 10%	100.0	101.9	106.3	115.5
q = 10%				
g = 0	100.0	95.5	92.1	91.8
g = 5%	100.0	100.3	101.6	106.2
g = 10%	100.0	105.1	111.5	122.1
q = 20%				
g = 0	100.0	98.0	95.5	95.5
g = 5%	100.0	102.9	105.3	110.5
g = 10%	100.0	107.8	115.6	127.1
q = 30%				
g = 0	100.0	100.1	98.3	98.4
g = 5%	100.0	105.1	108.3	113.9
g = 10%	100.0	110.1	118.9	131.0

Note: AGEIM is (25, 50, 25) in all cases where there is immigration.

Table 6. Simulations for Alpha Assuming Alternative Fertility Rates (F) with No Immigration

	t = 0	t = 1	t = 2	t = 3
		------------------- F = 1.6 -------------------		
Population	100.0	104.5	98.0	87.1
- growth rate	--	4.5	-6.2	-11.1
- proportion old	16.8	26.6	31.5	31.9
- proportion child	25.7	21.9	20.2	20.3
LF/Pop. Ratio	48.1	44.5	42.2	41.7
National income	100.0	96.8	86.1	75.6
- per capita	100.0	92.6	87.9	86.8
- wtd. per capita	100.0	90.7	85.2	84.2
		------------------- F = 2.0745 -------------------		
Population	100.0	111.3	110.6	111.4
- growth rate	--	11.3	-0.6	0.8
- proportion old	16.8	25.0	27.9	25.0
- proportion child	25.7	26.7	23.2	26.6
LF/Pop. ratio	48.1	41.8	42.3	41.5
National income	100.0	96.8	97.3	96.1
- per capita	100.0	87.0	88.0	86.3
- wtd. per capita	100.0	87.5	86.7	86.7
		------------------- F = 2.5 -------------------		
Population	100.0	117.3	121.9	135.9
- growth rate	--	17.3	3.9	11.5
-proportion old	16.8	23.7	25.3	20.5
- proportion child	25.7	30.5	25.4	31.7
LF/Pop. ratio	48.1	39.6	42.3	40.5
National income	100.0	96.8	107.4	114.6
- per capita	100.0	82.4	88.1	84.3
- wtd. per capita	100.0	84.8	87.9	87.3

Note: See relevant parts of note to Table 1. F = 2.0745 is the natural replacement rate.

Table 7. Simulations for Alpha, with and without Immigration, Allowing for Declining Mortality Rates and Increased Labour Force Participation of Seniors

		No immigration			Immigration, q = 20%		
	t = 0	t = 1	t = 2	t = 3	t = 1	t = 2	t = 3
		------------------------------- constant mortality ---------------------------------					
Population	100.0	104.5	98.0	87.1	125.4	151.1	177.4
- growth rate	--	4.5	-6.2	-11.1	25.4	20.5	17.4
- proportion old	16.8	26.6	31.5	31.9	22.2	23.3	24.1
- proportion child	25.7	21.9	20.2	20.3	22.4	22.8	22.1
LF/Pop. ratio	48.1	44.5	42.2	41.7	47.1	45.9	45.9
National income	100.0	96.8	86.1	75.6	122.8	144.3	169.4
- per capita	100.0	92.6	87.9	86.8	98.0	95.5	95.5
- wtd. per capita	100.0	90.7	85.2	84.2	96.2	93.9	93.6
		------------------------------ declining mortality ---------------------------------					
Population	100.0	106.5	103.3	94.7	127.8	158.8	192.6
- growth rate	--	6.5	-3.0	-8.4	27.8	24.2	21.3
- proportion old	16.8	27.6	34.3	36.3	23.0	25.3	27.0
- proportion child	25.7	21.5	19.3	18.8	22.1	22.0	21.1
LF/Pop. ratio	48.1	44.1	40.9	39.5	46.7	45.0	44.5
National income	100.0	97.7	87.8	77.8	124.3	148.6	178.3
- per capita	100.0	91.7	85.0	82.2	97.2	93.6	92.6
- wtd. per capita	100.0	89.6	82.0	79.1	95.3	91.7	90.2
		-------------------- declining mortality, increased LFP ----------------------					
Population	100.0	106.5	103.3	94.7	127.8	158.8	192.6
- growth rate	--	6.5	-3.0	-8.4	27.8	24.2	21.3
-proportion old	16.8	27.6	34.3	36.3	23.0	25.3	27.0
- proportion child	25.7	21.5	19.3	18.8	22.1	22.0	21.1
LF/Pop. ratio	48.1	45.8	42.7	41.3	48.1	46.4	46.0
National income	100.0	101.4	91.7	81.4	128.0	153.2	184.2
- per capita	100.0	95.2	88.8	86.0	100.1	96.5	95.6
- wtd. per capita	100.0	93.0	85.6	82.7	98.1	94.5	93.2

Note: See relevant parts of note to Table 1. AGEIM is (25, 50, 25) when there is immigration. Declines in mortality are at the average group-specific percentage rates of decrease per generation over the previous three-generation time span. Increased LFP means Seniors' labour force participation rates are increased by half (from 20% to 30% for men, 10% to 15% for women).

Table A1. The Q Matrix for a Stable Alpha Population with Calibrated Survival Rates

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10
Row 1	0	$s_{f0}r_fF$	0	0	0	0	0	0	0	0
Row 2	0.9942	0	0	0	0	0	0	0	0	0
Row 3	0	0.9769	0	0	0	0	0	0	0	0
Row 4	0	0	0.8635	0	0	0	0	0	0	0
Row 5	0	0	0	0.3798	0	0	0	0	0	0
Row 6	0	$s_{m0}r_mF$	0	0	0	0	0	0	0	0
Row 7	0	0	0	0	0	0.9875	0	0	0	0
Row 8	0	0	0	0	0	0	0.9617	0	0	0
Row 9	0	0	0	0	0	0	0	0.785	0	0
Row10	0	0	0	0	0	0	0	0	0.2575	0

Note: sf0 = .9940, sm0 = .9924.