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OBJECTIVE

The paper explores the population effects of male preference stopping rules and of alternative combinations of fertility rates and male-biased birth sex ratios.

METHODS

The "laboratory" is a closed, stable population with five age groups and a dynamic process represented by a compact Leslie matrix. The new element is sex-selective abortion. We consider nine stopping rules, one with no male preference, two with male preference but no abortion, and six with male preference and the availability of abortion to achieve a desired number of male births. We calculate the probability distribution over the number of births and number of male births for each rule and work out the effects at the population level if the rule were adopted by all women bearing children. We then assess the impact of alternative combinations of fertility rates and male-biased sex ratios on the population.

RESULTS

In the absence of sex-selective abortion, stopping rules generally have no effect on the male/female birth proportions in the population, although they can alter the fertility rate, age distribution, and rate of growth. When sex-selective abortion is introduced the effect on male/female proportions may be considerable, and other effects quite different as well. The contribution of this paper is the quantification of effects that might have been predictable in general but which require model-based calculations to see how large they could be. As the paper shows, they could in fact be very large; a population in which sex-selective abortion was widely practised could look quite different from what it would otherwise be.

1. INTRODUCTION

The theoretical consequences of sex-preference stopping rules at the family level have been known for a long time – in particular, the lack of any effect on the overall proportions of male and female births when the probabilities for individual births are fixed and the same throughout the population (Goodman, 1961, Keyfitz, 1968, and others). However, there is an accumulation of evidence now to indicate the use of abortion to alter those probabilities in favour of male births in countries where male preference is common; see Bongaarts (2013) for a recent survey of evidence of male preference and the use of sex-selective abortion. Some countries of Eastern and Southern Asia have received particular attention in that regard (see, for example, Guilmato, 2010, Jiang, Li, and Feldman, 2011) and there is evidence of the use of abortion by emigrants from those countries who are resident elsewhere: Dubuc and Coleman (2007), Almond and Edlund (2008), Abrevaya (2009), Almond, Edlund, and Milligan (2013), Ray, Henry, and Urquia (2012). (While male preference has received most of the attention in the literature, including the present paper, female preference is certainly possible also; see Fuse, 2013, for evidence of that from Japan.) Yamaguchi (1989) explored the effects of stopping rules on birth order and number of siblings in the absence of direct parental control over the sex probabilities. More recently, Yadava, Kumar, and Srivistava (2013) investigated the effects on the sex probabilities at birth of stopping rules when selective abortion is an option. The question on which we focus here is a "what if" question: What if there were a change in birth probabilities at the individual family level; how would that translate into changes in the characteristics of the population?

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This is an exploratory paper. We explore, in particular, the potential effects of male preference stopping rules on a stable population, and more generally, the implications of higher male/female ratios at birth. We begin by choosing a set of nine stopping rules, three with no abortion, six with, and derive the associated probability functions at the individual family level. We simulate the consequences of each rule for the population as a whole – in particular, its rate of growth and age and sex distributions, were the rule to be adopted throughout the population. We then move away from the idea of explicit stopping rules, specify nine alternative sex ratios at birth (assuming unspecified rules or mixtures of rules underlying them), couple the ratios with alternative fertility rates, and derive the stable population growth rates and sex distributions that would result.

The instrument that we use to simulate aggregate effects is a compact Leslie matrix representing an artificial population with two sexes and broad age groups. The matrix is calibrated with realistic survival rates, allows the insertion of alternative combinations of fertility rates and sex ratios at birth, as required for particular simulations, and can be used easily to derive the resultant stable populations. We note and discuss the properties of the matrix as a prelude to its application in the simulations. To emphasize that our population is artificial we think of it as being the population of a mythical country Alpha, as we shall call it. The population of Alpha then provides a laboratory in which to explore the population implications of family preferences for male children.

2. STOPPING RULES

We consider the nine rules that a family might adopt. (The choice is somewhat arbitrary but provides a wide range of population outcomes for analysis.) One of the rules assumes no male preference. Two others reflect male preference but no effective way of altering the probabilities of a particular male or female birth. The remaining ones allow for the possibility of knowing the sex of a child at an early stage of pregnancy and using selective abortion to increase the probability that the next birth will be a male. In specifying the rules we abstract from miscarriages and stillbirths, and assume that in the absence of sex selective abortion a fetus would proceed to a live birth. We abstract also from the possibility of multiple births; all births are singletons. We label the stopping rules S0, S1, …, S8.

The rules fall into four categories. They differ with regard to male preference, preferences for family size, whether abortion is permitted and, if so, whether its use is limited or unlimited. Rule S0 is in category 1: it assumes no male preference and serves as a reference rule with which to compare the effects of the subsequent ones. Rules S1 and S2 fall into the second category: they assume male preference but no abortion option; S1 has a maximum number of births of three, S2 a maximum of four. Rules S3 and S4 fall into the third category: they assume male preference with limited use of abortion and maximum numbers of births of three and four, respectively. Rules S5 – S8, which make up the fourth category, assume male preference with unlimited use of abortion: they have maxima of three, four, five, and six births, in that order. The main distinguishing feature of this latter category is that unlimited abortion

allows exact determination of the number of males. The precise definitions of the rules are as follows.

S0: There is no male preference: stop only when the number of children ever born is three.

S1: Stop when the first male child is born or when the total number of children ever born is three, whichever comes first.

S2: Stop when the second male child is born or when the number of children ever born is four, whichever comes first.

S3: Stop when the first male child is born. If there have been two births and no males, check the sex of the next fetus and abort if female. Allow the third birth to take place only if a fetus is male or there have been three successive abortions of female fetuses. The third birth will then be either male (with high probability) or female, and the three births will include one or no males.

S4: Stop when the second male child is born. If there have been three births and one or no males, check the sex of the next fetus and abort if female. Allow the fourth birth to take place if the fetus is male or there have been three successive abortions of female fetuses. The fourth birth will then be either male (with high probability) or female, and the four births will include two, one, or no males.

S5: Permit no more than one female birth; abort additional female fetuses, with no limit on the number of abortions. Stop when there are two male births.

S6: Permit no more than one female birth; abort additional female fetuses, with no limit on the number of abortions. Stop when there are three male births.

S7: Permit no more than two female births; abort additional female fetuses, with no limit on

the number of abortions. Stop when there are three male births.

S8: Permit no more than two female births; abort additional female fetuses, with no limit on

the number of abortions. Stop when there are four male births.

The joint probability functions for number of births (n) and number of male births (m) for

these stopping rules are as follows, with p the probability of a male birth (assumed

independent of parity), q = 1 – p the probability of a female birth, and a the probability of an

abortion, which is set equal to the probability of a female fetus (a = q, but there is no birth); it is

assumed, in the absence of abortion, that any fetus would survive to become a live birth. We

put abortion functions in square brackets and place them in the probability expressions in that

form to indicate their position in the sequence of births.

Rule S0: $P_0(n,m) = \binom{n}{m} p^m q^{3-m}$ (for n = 3, m=0,1,2,3)

Rule S1: $P_1(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 1,2,3, m= 1)

$$= q^3$$ (for n = 3, m = 0)

Rule S2: $P_2(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 2,3,4, m = 2)

$$= 4pq^3$$ (for n = 4, m = 1)

$$= q^4$$ (for n = 4, m = 0)

Rule S3: $P_3(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 1,2, m = 1)

$$= q^2[1+a+a^2+a^3]p$$ (for n = 3, m = 1)

$$= q^2[a^3]q \quad \text{(for n = 3, m = 0)}$$

Rule S4: $P_4(n,m) = \binom{n-1}{n-m}p^m q^{n-m}$ (for n = 2,3, m = 2)

$$= 4q^3[1+a+a^2+a^3]p \quad \text{(for n = 4, m = 1)}$$

$$= 3pq^2[1+a+a^2+a^3]p \quad \text{(for n = 4, m = 2)}$$

$$= q^3[a^3]q \quad \text{(for n = 4, m = 0)}$$

Note: Rules S5 to S8 allow an unlimited number of abortions of unwanted female fetuses, and hence (in our theoretical framework) a desired male birth with certainty. In what follows, we use the symbol [1] to indicate such a male birth with probability 1, as distinguished from a natural birth with probability p.

Rule S5: $P_5(n,m) = p^2$ (for n = 2, m = 2)

$$= q[1][1] + pq[1] \quad \text{(for n = 3, m = 2)}$$

Rule S6: $P_6(n,m) = p^3$ (for n = 3, m = 3)

$$= q[1][1][1] + pq[1][1] + p^2q[1] \quad \text{(for n = 4, m = 3)}$$

Rule S7: $P_7(n,m) = p^3$ (for n = 3, m = 3)

$$= 3qp^3 \quad \text{(for n = 4, m = 3)}$$

$$= q^2[1][1][1] + 2pq^2[1][1] + 3p^2q^2[1] \quad \text{(for n = 5, m = 3)}$$

Rule S8: $P_8(n,m) = p^4$ (for n = 4, m = 4)

$$= 4qp^4 \quad (\text{for } n = 5, m = 4)$$

$$= q^2[1][1][1][1] + 2q^2p[1][1][1] + 3q^2p^2[1][1] + 4q^2p^3[1] \quad (\text{for } n = 6, m = 4)$$

The numerical values of these probabilities are provided in Table 1. Cells with no entries represent impossible combinations of n and m, under the specified stopping rules, and show how the rules restrict the numbers of births and the male/female combinations. Also shown in the table are E(n) and E(m) (the expected numbers of births and male births), the proportion r = E(m)/E(n), and the corresponding male/female odds ratio, m/f = E(m)/(E(n)-E(m)). The probabilities assume m/f = 1.05 for an individual birth in the absence of sex selection - a ratio common to many countries (U.S., U.K., Canada and others) - and that is also the overall ratio calculated from the expected values for stopping rules S0, S1, and S2. With a limited possibility of sex selection through abortion (maximum three times) the male/female ratio rises to 1.3 or 1.4 (stopping rules S3, S4). With unlimited use of the abortion option (rules S5 to S8) it goes as high as 3.5 (in rule S6). (The assumption of an unlimited number of abortions is a matter of theoretical convenience. A more realistic interpretation would be that the number of abortions is limited but with a maximum sufficient to drive the probability of a male birth close to 1.)

3. AN ARTIFICIAL POPULATION

We explore the potential implications for a population, in particular its age distribution, sex distribution, and rate of growth, if the above rules were to be adopted universally; beyond that, we explore, more generally, the aggregate effects of alternative combinations of fertility

rates and male/female birth distributions. For those purposes we specify an artificial population, one that is realistic in general form but simple enough to allow easy experimentation. To emphasize its artificiality we think of it as representing a mythical country Alpha, the population dynamics of which are defined by a two-sex Leslie matrix Q of dimension 10 x10. (It is convenient to think of Alpha as a country but it could be a region within a country, or it could be defined by a common religion, ethnicity, or culture.) There are five broad age groups recognized in Alpha, each of which can be viewed as consisting of 20 individual ages. For convenience we label the groups as follows: Children (ages 0 to 19), Young Adults (ages 20 to 39), Middle Aged (ages 40 to 59), Retired (60 to 79), and Old (ages 80 to 99), with no survivors at age 100. The Young Adult group is the fertile group in Alpha; women in that group bear all the children. (This simplification is convenient since it avoids having to deal with the age distribution of fertility rates, which is of little relevance for present purposes.) The population of Alpha provides a laboratory in which to ask what the aggregate effects would be of childbearing decisions made at the individual family level.

The first five rows of the Q matrix are for females, the last five for males. The Q(1,2) cell is calculated, for female babies, as $s_{f0}r_fF$, where F is the fertility rate for Young Adult females (the total fertility rate, since there is only the one childbearing age group), r_f is the proportion of females at birth, and s_{f0} is the survivor correction for female births (Kintner, 2008, p. 323). Correspondingly, for male babies, the Q(6,2) cell is calculated as $s_{m0}r_mF$. The values of r_m, r_f (= 1-r_m), and F are set experimentally, at various levels; s_{f0} and s_{m0} are parameters with fixed values.

The group-to-group survival rates are in the normal positions for a Leslie matrix: s_{f1} to s_{f4} in cells Q(2,1), Q(3,2), Q(4,3), Q(5,4) for females, s_{m1} to s_{m4} in cells Q(7,6), Q(8,7), Q(9,8), Q(10,9) for males. We draw on 2001 Canadian life tables for calibration of the survival rates; the rates are derived from the L_x values in those tables. (The Canadian life tables are based on mortality data for the three years 2000, 2001, and 2002 but are commonly referred to as 2001 tables; see Statistics Canada, 2006.) The Q matrix itself is presented in Table 2, with the survival rates shown in numerical form, as calibrated values, and the fertility elements (which vary from experiment to experiment) in symbolic form.

Each age group in the Alpha population consists of 20 years; correspondingly, the time interval can be thought of as 20 years, and referred to as a *generation*. Now let X_0 be a column vector representing the population at time 0. With Q fixed the population k generations later is given by $X_k = Q^k X_0$. For an arbitrary initial specification of X_0 the population can be converted to stable form by letting k increase until there is no further change in the proportionate age distribution. This provides a convenient procedure for simulating the effects of different specifications of F and r_f. (The Perron-Frobenius theorem, as adapted to a Leslie matrix, ensures full ergodicity in the sense of nondegenerate convergence to a stable population with growth rate independent of the initial population vector, as long as the first two elements of the vector are not both zero. The age/sex structure of the stable population is also independent of the initial vector – in the sense discussed in the next section – for a sufficiently wide range of choices to make the process ergodic in that regard too, for practical purposes; any reasonable choice of an initial vector will do. See Cull and Vogt, 1973, Cohen, 1979, and Keyfitz and Caswell, 2005, chapter 7.)

4. PROPERTIES AND IMPLICATIONS OF THE Q MATRIX

The Q matrix has the following feature: the stable population form that it generates is cyclical. The matrix is imprimative, with index of of imprimativity 2, and has exactly two real nonzero eigenvalues, equal in value but of opposite sign (Keyfitz and Caswell, 2005). The cycle is two generations in length so that if stability is achieved the proportionate age distributions for X_k and X_{k+2} are identical, and similarly for X_{k+1} and X_{k+3}. Only if the stable population is also stationary does the cycle disappear, making both the sizes and the proportionate age distributions at k and k+1 the same (the nonzero eigenvalues are then 1 and -1). In the general case, to put it differently, the stable form of the population encompasses, in a 2-period sequence, both a birth effect and a subsequent echo effect resulting from the entry of last generation's newborn children into the childbearing Young Adult group this generation. While the proportionate age distributions behave cyclically the average of every two consecutive age distributions is strictly stable, and following Cull and Vogt (1973), that is what we use in analysing stable population age distributions. The same is true of sex distributions.

The cyclical characteristic has implications for the calculation of growth rates also. Let X_t be the population vector in the (cyclically) stable growth state at time t. We then have $X_{t+2} = Q^2 X_t = (1+G)X_t$ and $X_{t+3} = Q^2 X_{t+1} = (1+G)X_{t+1}$, where G is the two-generation rate of growth. The average one-generation (20-year) growth rate is then $(1+G)^{1/2} - 1$ and the average annual rate is $g = (1+G)^{1/40} - 1$. As it is common to think of population growth in terms of annual rates we show the g values in reporting aggregate results below.

The link between the growth rate and the fertility rate and male/female ratio at birth can be established as follows. Let the elements of X_t be labeled X_{ti}, i = 1,…,10. Based on X_{t+2} = $Q^2 X_t$ = $(1+G)X_t$, the first element of X_{t+2} (Children) is then $X_{t+2,1} = s_{f1}s_{f0}r_f F X_{t1} = (1+G)X_{t1}$, and hence g can be calculated as $(s_{f1}s_{f0}r_f F)^{1/40}$ - 1. Note that, for a given fertility rate and proportion of female births, the rate of growth depends entirely on the survival rates for female children: s_{f0}, the survivor correction for female births, and s_{f1}, the rate that determines the proportion of female children in one generation who survive to bear children in the next. All other survival rates are irrelevant for the growth rate. (That is a well-known result for a Leslie matrix; we note it here for use later in the paper.)

5. POPULATION EFFECTS OF THE STOPPING RULES

The effects of each of the nine family stopping rules on the stable population of Alpha are displayed in Table 3, assuming in each case that the rule is adopted throughout the population by all families that have children. The overall proportions of male births and the corresponding male/female ratios are shown at the top of the table, along with the fertility rates and annual population growth rates. It is assumed that the proportion of women who bear no children is 15 percent in Alpha so that a fertility rate is calculated as .85 times the corresponding E(n) value in Table 1. (The choice of .85 for our fictitious country Alpha is arbitrary. For comparison, the proportion of women 40 – 44 in the United States who had given birth was reported as .82 for 2008, .90 for 1976, based on data from the Current Survey of Population; see Livingston and Cohn, 2010.) Otherwise it is assumed that all families adopt the

same stopping rule. Also on display in the table are the age distributions of the population (averages over the stable two-period cycle) and the proportions of males in the five age groups.

The effects of a stopping rule on the population depend on the preferred number of male births incorporated into the rule as a goal, the "aggressiveness" with which the goal is pursued (the total number of births permitted in pursuit of it), and whether or not an abortion option is available. Rule S0 has no male preference goal, simply an overall fixed number of children, set at three. Rule S1 has a goal of one male birth, a willingness to allow up to three births to achieve that goal, and no abortion option. Rule S2 is similar but the goal is now two males, with up to four births permitted. As expected, the population sex ratio at birth is unaffected by the adoption of either S1 or S2: the male/female ratio at birth is 1.05 for each of the S0, S1, and S2 rules. (This result has been well known since the early work of Goodman, 1961, Keyfitz, 1968, and others.) However, the population fertility rate, growth rate, and age distribution can be very much affected by the choice of a rule, even if the sex ratio is not. S0, with its fixed three births per family, produces a population fertility rate of 2.550, well above the natural replacement rate (which is a little under 2.1), and a corresponding positive annual rate of growth of .517 percent. S1, with its stop-at-the-first-male restriction, lowers the fertility rate to 1.467 and induces a negative rate of growth, -.862 percent per annum. S2, on the other hand, has the opposite effect, pushing the fertility rate up to 2.736 and the growth rate well into the positive range, .695 percent. The sex ratio of the population is of course the same under each of the three rules but the age distribution varies greatly: in S1, 18.6 percent of the population are in the Children category, 9.4 percent in the Old category; in S3, 30.7 percent are Children, 4.5 percent are Old. Thus the introduction of a stopping rule that leaves the sex ratio

at birth unchanged can have major effects on other characteristics of the population. This too is not a surprising result but the calculations for rules S0, S1, and S2 in Table 3 provide some indication of the possible sizes of the effects for rules that have no abortion option.

The abortion option, the new element in the calculation of stable population effects, is introduced in stopping rules S3 to S8. The calculations in Table 3 for these rules indicate the possibilities for increasing the male/female ratio at birth by the use of this option, beyond the natural ratio of 1.05 that applied under the earlier rules. S3 and S4 impose a limit of three on the number of abortions allowed, S5 to S8 impose no limit.

The smallest increase in the male/female birth ratio among the abortion-permitted rules comes about under S3 (an increase from 1.05 to 1.335), the largest comes about under S8 (an increase to 2.315). S3 induces a large negative rate of growth (-1.184 percent per year), S8 a large positive rate (.936 per year). Effects on the overall sex distribution of the population are, of course, pronounced: the proportion of males over the full set of rules in Table 3, all ages combined, varies from 49.1 percent (S1) to 76.1 percent (S6). Effects on the age distribution are also significant: the range for Children is 16.5 percent of the total population (S5) to 32.9 percent (S8); the range for the Old group is 3.6 percent (S8) to 10.6 percent (S3). That a male preference stopping rule with sex selective abortion can (in theory) negate the growth effects of even a quite high fertility rate is well illustrated by S6: the fertility rate is 3.286 but the stable population is in a state of decline, at -.793 percent per annum.

We offer three summary generalizations at this point. The first is that universal adoption of a male preference stopping rule imposed at the family level can have major effects on the

characteristics of a population, which is obvious, but the magnitudes of those effects and their interrelationships are not obvious without an exercise such as the present one. Secondly, sex selective abortion as an element of a stopping rule (and the degree of limitation imposed on its frequency of use) can affect greatly the impact of the rule on the population; among other things, it can alter the sex ratio at birth, which would not otherwise happen at the population level. Thirdly, the effects that a given stopping rule can have on the proportion of children in a stable population results from the combination of its effect on the fertility rate and its effect on the proportion of females within the fertile age range, and those two effects may differ in the presence of sex-selective abortion. One consequence is a possible combination of high fertility and little or no population growth, or even negative growth.

6. INTERACTION OF THE FERTILITY RATE AND THE SEX RATIO AT BIRTH

Now imagine that there is some unspecified set of stopping rules or mixture of stopping rules that prevails in the population of Alpha, with possible sex-selective abortion incorporated into the rules. Thus we move away from the idea of specifying particular rules and working out their implications, and simply consider alternative possible combinations of fertility rates and sex ratios at birth that might have resulted from such an unspecified set. (The ability to terminate pregnancies because of sex, coupled with possible heterogeneity of family choice of a rule across the population, would make virtually any such combination theoretically possible, over a range even wider than what we now explore.) We focus in particular on the growth rate and the overall proportion of males in the stable population of our fictitious country Alpha, selecting nine alternative fertility rates, from 1.0 to 5.0, and coupling them with nine alternative

male/female (m/f) ratios, from 1.05 to 1.45. The results are presented in Table 4. The choice of

fertility rates represents, in a rough way, a range of rates observed among countries in recent

decades. The m/f ratios are a convenient choice for exploring the effects of variations in those

ratios: the lower bound, 1.05, is a commonly observed ratio; the upper bound, 1.45, is an

arbitrary value chosen for exploratory purposes – a very high value in comparison with ratios

actually observed, although not as high as some of the hypothetical rule-generated ratios in

Table 3.

Results for the 81 combinations of fertility rates and m/f ratios are shown in Table 4.

The effect of increasing the sex ratio at birth in favour of males is to reduce the rate of growth

and increase the proportion of males (decrease the proportion of females) in the population,

for any given fertility rate. With a fertility rate of 2.5 children per woman and an m/f ratio of

1.05, for example, the population of Alpha grows at a rate of .468 percent per year; if the m/f

ratio is increased to 1.3, the growth rate falls to .179 percent, and if the ratio is allowed to

increase further, to 1.45, the rate falls to .021 percent, almost no growth at all. To put it

differently, the natural replacement fertility rate – the rate required to produce a stationary

population - is a little under 2.1 children per woman with the commonly observed m/f ratio

1.05; with a very high ratio of 1.45 the natural replacement rate is about 2.5 children per

woman. The proportion of males in the overall population ranges from 48.4 percent at one

extreme (F = 1.0, m/f = 1.05) to 58.4 percent at the other (F = 5.0, m/f = 1.45).

We observe that while a country with a very high fertility rate, by international

standards, and accordingly a very high rate of population growth, would have its growth rate

reduced if the m/f ratio were to increase, it could still have a high positive rate of growth, within the bounds imposed on the ratio in Table 4 (and indeed much beyond those bounds). If Alpha had a fertility rate of 5.0, for example, and an m/f ratio of 1.05, its population would be growing at 2.224 percent per year, or about 25 percent over a decade. If its fertility rate were to remain the same, but its m/f ratio were to increase to as high as 1.45, its growth rate would fall to 1.769 per year, or about 19 percent per decade - a substantial decrease, certainly, but still a far distance from zero growth. Going back to Table 3, even stopping rule S8, with its fertility rate of 4.869 and an m/f ratio of 2.315, would produce a population growth rate of .936 per year (roughly 10 percent per decade). It is of course possible to imagine (in theory) a stopping rule that would combine with a fertility rate of 5.0 to produce a declining population but the associated m/f ratio would have to be extremely high. (A limiting case would be a rule that required *all* female fetuses to be aborted, in which case the population would vanish completely within five generations.) To obtain zero or negative growth rates would require much lower fertility rates and/or much higher m/f ratios than the ones just noted in the tables.

7. THE TRADE-OFF BETWEEN FERTILITY AND THE SEX RATIO AT BIRTH

The stable population rate of growth can be thought of as determined by two parameters, the fertility rate and the proportion of female births (conditional on the early-life female mortality rates s_{f0} and s_{f1}). Manipulating the equation for the growth rate in section 4 we may write it in the alternative form $\ln(1+g) = H + .025 \ln F + .025 \ln r_f$, where the constant H = .025 ($\ln s_{f0} + \ln s_{f1}$). The possibilities for trade-off between fertility and the female birth proportion are implicit in Table 4 but this form of the equation makes them explicit: for any

given value of g the pairs of F and r_f that would yield that rate can be calculated. Trade-off curves of this kind are shown in Figure 1 for g = -1%, 0, and 1%, for illustration. The horizontal axis in the figure represents F, the vertical axis r_f, and the points on a curve the alternative combinations of F and r_f that would yield the given growth rate. A stationary population (g = 0), for example, could be supported by a fertility rate of 2.1 coupled with a female birth ratio of .483. But it could also be supported by a fertility rate of 1.6 coupled with a ratio of .634, or by a fertility rate of 3.0 coupled with a ratio of .338. The trade-off curves shift to the right as the growth rate increases but of course retain their curvature.

8. ALTERNATIVE SURVIVAL RATES AND THE RATE OF GROWTH

We have calibrated the Q matrix by assigning to it a particular set of survival rates. A different set would affect the results rather little. The results that we derive for a stable population depend critically on the fertility rate and the male/female proportions at birth; they are relatively insensitive to the choice of survival rates. We illustrate that now by recalculating some of the stable population growth rates in Table 4 with death rates increased by multiples of two, five, and ten, and the associated survival rates reduced accordingly.

As shown earlier, the annual population growth rate is given by g = $(s_{f1}s_{f0}r_fF)^{1/40} - 1$ so that the only two survival rates that affect the stable growth rate are s_{f0} and s_{f1}. We now lower these survival rates by assuming that the corresponding death rates increase by a factor K. A survival rate s is thus replaced by s* = 1 − K(1 - s). We choose four alternative K values: K = 1, 2, 5, 10. Both survival rates are adjusted in this way and the resulting growth rates are shown in Table 5 for alternative combinations of m/f and F. To show the limits of the effects of the

recalculations we choose the combinations at the four corners of the growth rate panel in Table 4, the four combinations of m/f = 1.05 or 1.45 with F = 1.0 or 5.0, that is. As in Table 4, the growth rates are expressed in percentage form.

Setting K = 1 means no change in survival rates, and hence stable population growth rates that are identical to those in Table 4. A two-fold increase in the death rates (K = 2) changes the growth rates by no more than .031 percent, a five-fold increase (K = 5) by no more than .123 percent. Even a ten-fold increase (K = 10) changes the rates at most by .281 percent. The growth rates in Table 5 thus show very little sensitivity to changes in survival rates.

9. SUMMARY AND DISCUSSION

The purpose of the paper has been to explore the effects of male preference stopping rules on the characteristics of a population, and more generally the population effects of alternative combinations of fertility rates and male-biased birth sex ratios. The "laboratory" for the exploration is a mythical country Alpha with a closed, stable population, five age groups, and a dynamic process represented by a compact Leslie matrix. The new element in the analysis is sex-selective abortion. In the absence of sex-selective abortion, stopping rules have no effect on the male/female proportions; in the presence of sex-selective abortion the effect on the male/female proportions may be large, and other effects quite different from what they would otherwise be.

We began with a particular set of nine stopping rules, one with no male preference, two with male preference but no abortion, and six with male preference and the availability of abortion as an aid to achieving a desired number of male births. We calculated the probability

distribution over the number of births and number of male births for each rule. We then assumed, for each, that it was adopted by all women bearing children, and worked out the effects that that would have on the population – in particular the effects on the male/female ratio at birth, the overall fertility rate, the age distribution of the population, the sex distribution at each age, and the rate of growth. We then changed course: we assumed that there were unspecified combinations of stopping rules that could generate particular fertility rates and male-biased sex ratios at the aggregate level, and calculated the effects that that would have on the population for a large number of combinations of the two. (Given a wide range of possible stopping rules, and potential heterogeneity in the choice of a rule within the population, there could in fact be a wide range of fertility/sex-ratio outcomes within our framework of analysis.)

There are several observations of a general nature that one can make, based on the calculations in the paper. The first is obvious and well known: universal adoption of a stopping rule when there is no ability to affect the probability that an individual birth will be of a given sex can affect the fertility rate, age distribution, and rate of growth of the population, but not its sex distribution. The second observation is that the introduction of sex selective abortion as an option in family planning makes it possible for a stopping rule (or combination of rules) to affect the sex distribution of births at the aggregate level, and hence the sex distribution at every age in a stable population; in particular, it would allow a societal preference for male children to manifest itself and have a major impact on the characteristics of the population. (As illustration of what "skewed sex ratios at birth" can do to actual populations, see Guilmato, 2010, with regard to China and India.) The third observation is that the influence of a male

preference stopping rule on the population can be thought of as being determined by (a) the targeted number of male children, (b) the "aggressiveness" built into the stopping rule, represented by the number of births that would be accepted in trying to achieve the target, (c) the availability of abortion as a strategic element in the application of the rule, and (d) the frequency with which abortion would be permitted in applying the rule. The fourth observation is that the number of births is, of course, determined jointly by the fertility rate and the proportion of females in the fertile age range, and hence by the sex ratio at birth in the previous generation, which affects the number of fertile women in the present one. The population growth rate is thus similarly determined – or, to put it differently, the same growth rate can be achieved by alternative combinations of the fertility rate and birth sex ratio, as seen in the trade-off curves of Figure 1. The fifth observation is that the rate of population growth, in particular, is quite insensitive to the specification of survival rates, and in fact only the early-life rates for females matter (s_{f0}, s_{f1}). We calibrated our model with a given set of survival rates but changing those rates has little effect on the growth rate; growth is driven almost entirely by the fertility rate and male/female ratio at birth in our experiments.

It is not surprising that the ability to change the birth probabilities in favour of males would have the types of effects just mentioned, if one thinks about the matter. What we would like to be viewed as the contribution of this paper is the analytical quantification of effects that might have been predictable in general, but which require model-based calculations to see how large they could in fact be. That has been our aim. In terms of the existing literature, we would like to think of the paper, with its emphasis on population effects, as a useful complement to

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the growing literature on sex-selective abortion and stopping rules at the family level, as noted in our introduction.

It is perhaps appropriate, in conclusion, to take note of some of the issues that we have not attempted to address. One thing the paper does not do is consider the dynamics of adjustment – the process of moving from one stable state to another in a situation in which sex selection becomes more widely practised. Nor does it consider how an actual (as opposed to theoretical) population might evolve in such a situation, or the population implications of migration from a country or region in which sex selection is more common to one in which it is not. (See in that regard our references at the beginning of the paper.) How an increased ratio of males to females in the general population might itself come to influence future fertility choices is another question that could be put on the list, and more generally the societal consequences of an increased proportion of men in the population, and a decreased proportion of women. At an analytical level, we have assumed homogeneity of fecundity in specifying stopping rules, and common male/female probabilities at birth in the absence of sex-selective abortion, thus ignoring natural differences that may in fact exist in a population. Also, we have disregarded the role that child mortality might play in the application of stopping rules – the death of a son, say, which might alter preferences in regard to subsequent births. One can think too of other matters of possible relevance that play no role in our analysis. Having said that, we observe that all papers ignore some things in order to focus on others – and so it is with ours.

10. ACKNOWLEDGEMENTS

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Figure 1. Trade-off Curves for Alternative Population Growth Rates

g = 1%

g = 0

g = - 1%

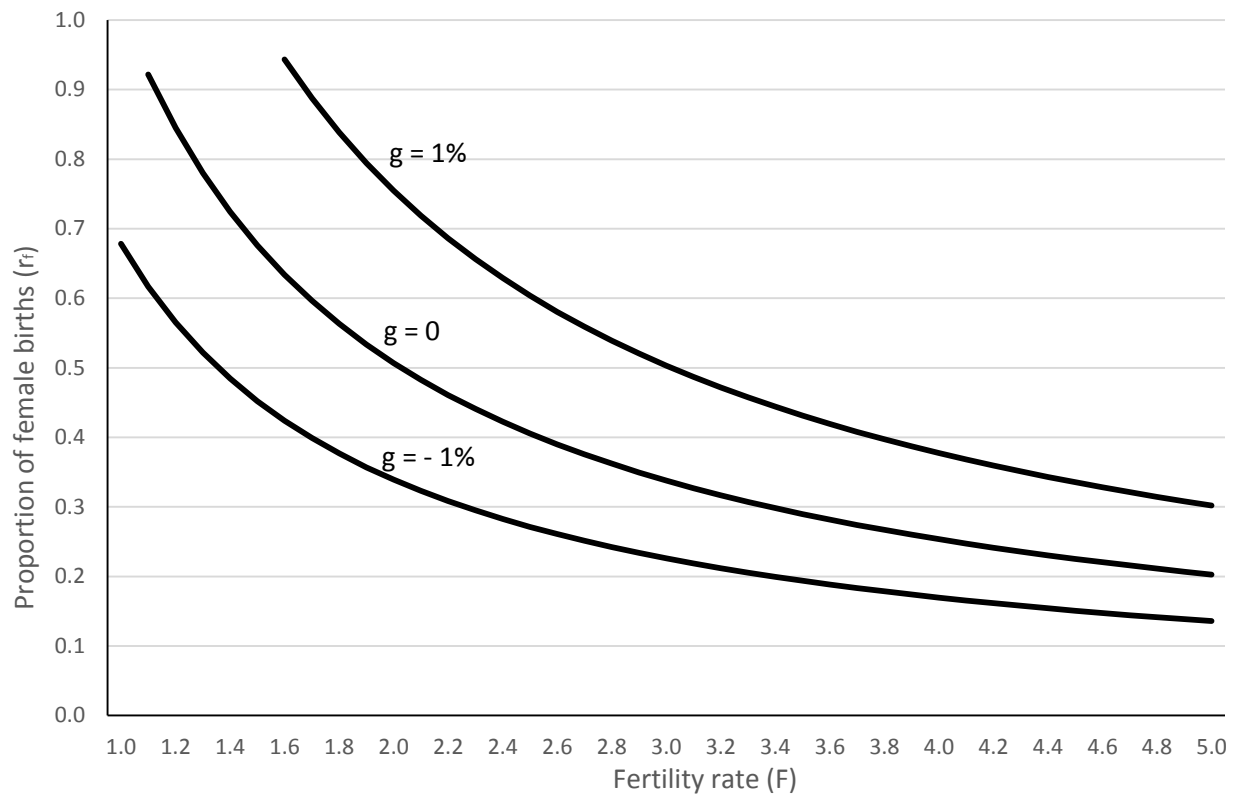Proportion of female births (r_f)

Fertility rate (F)

Table 1. Probability Distributions and Related Statistics at the Family Level: Numbers of
Births and Numbers of Male Births Under Alternative Stopping Rules

Number of births (n)	Number of males (m)	Prob(n,m) under given stopping rule								
		S0	S1	S2	S3	S4	S5	S6	S7	S8
1	0	--	--	--	--	--	--	--	--	--
1	1	--	0.512	--	0.512	--	--	--	--	--
2	0	--	--	--	--	--	--	--	--	--
2	1	--	0.250	--	0.250	--	--	--	--	--
2	2	--	--	0.262	--	0.262	0.262	--	--	--
3	0	0.116	0.116	--	0.013	--	--	--	--	--
3	1	0.366	0.122	--	0.224	--	--	--	--	--
3	2	0.384	--	0.256	--	0.256	0.738	--	--	--
3	3	0.134	--	--	--	--	--	0.134	0.134	--
4	0	--	--	0.057	--	0.007	--	--	--	--
4	1	--	--	0.238	--	0.130	--	--	--	--
4	2	--	--	0.187	--	0.345	--	--	--	--
4	3	--	--	--	--	--	--	0.866	0.197	--
4	4	--	--	--	--	--	--	--	--	0.069
5	0	--	--	--	--	--	--	--	--	--
5	1	--	--	--	--	--	--	--	--	--
5	2	--	--	--	--	--	--	--	--	--
5	3	--	--	--	--	--	--	--	0.669	--
5	4	--	--	--	--	--	--	--	--	0.134
5	5	--	--	--	--	--	--	--	--	--
6	0	--	--	--	--	--	--	--	--	--
6	1	--	--	--	--	--	--	--	--	--
6	2	--	--	--	--	--	--	--	--	--
6	3	--	--	--	--	--	--	--	--	--
6	4	--	--	--	--	--	--	--	--	0.797
E(n)		3.000	1.726	3.219	1.726	3.219	2.738	3.866	4.535	5.728
E(m)		1.537	0.884	1.649	0.987	1.857	2.000	3.000	3.000	4.000
E(m)/E(n)		0.512	0.512	0.512	0.572	0.577	0.731	0.776	0.662	0.698
m/f ratio		1.050	1.050	1.050	1.335	1.362	2.711	3.466	1.955	2.315

Note: In the absence of selective abortion a birth is assumed to be male with probability .5122.
A double dash indicates an impossible n,m combination.

Table 2. The Q Matrix for a Stable Alpha Population with Calibrated Survival Rates

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10
Row 1	0	$s_{f0}r_fF$	0	0	0	0	0	0	0	0
Row 2	0.9942	0	0	0	0	0	0	0	0	0
Row 3	0	0.9769	0	0	0	0	0	0	0	0
Row 4	0	0	0.8635	0	0	0	0	0	0	0
Row 5	0	0	0	0.3798	0	0	0	0	0	0
Row 6	0	$s_{m0}r_mF$	0	0	0	0	0	0	0	0
Row 7	0	0	0	0	0	0.9875	0	0	0	0
Row 8	0	0	0	0	0	0	0.9617	0	0	0
Row 9	0	0	0	0	0	0	0	0.7850	0	0
Row10	0	0	0	0	0	0	0	0	0.2575	0

Note: s_{f0} = .9940, s_{m0} = .9924.

Table 3. Stable Alpha Populations Resulting from Alternative Stopping Rules

	Stopping rule								
	S0	S1	S2	S3	S4	S5	S6	S7	S8
Proportion male births (r_m)	0.512	0.512	0.512	0.572	0.577	0.731	0.776	0.662	0.698
Male/female ratio at birth	1.050	1.050	1.050	1.335	1.362	2.711	3.466	1.955	2.315
Fertility rate (F)	2.550	1.467	2.736	1.467	2.736	2.327	3.286	3.855	4.869
Annual % growth rate	0.517	-0.862	0.695	-1.184	0.339	-1.189	-0.793	0.637	0.936
Population age distribution									
- children	29.2	18.6	30.7	16.5	27.8	16.7	19.5	30.4	32.9
- young adults	26.1	21.9	26.5	20.8	25.8	21.1	22.6	26.5	27.1
- middle aged	22.8	25.3	22.3	25.5	23.3	25.8	25.6	22.6	21.7
- retired	17.0	24.8	16.0	26.6	17.8	26.5	24.1	16.2	14.6
- old	4.9	9.4	4.5	10.6	5.2	9.9	8.1	4.3	3.6
- all ages	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Proportion males									
- children	51.2	51.2	51.2	57.1	57.6	73.0	77.6	66.1	69.8
- young adults	51.0	51.0	51.0	57.0	57.5	72.9	77.5	66.0	69.7
- middle aged	50.6	50.6	50.6	56.6	57.1	72.6	77.2	65.6	69.3
- retired	48.2	48.2	48.2	54.2	54.7	70.6	75.5	63.4	67.3
- old	38.7	38.7	38.7	44.5	45.0	62.0	67.6	54.1	58.2
- all ages	49.9	49.1	50.0	54.9	56.3	71.2	76.1	65.0	68.9

Note: The proportion of women bearing no children is set at 15% for this table.

Table 4. Annual Rate of Growth and Proportion of Males in a Stable Alpha Population with Alternative Combinations of Fertility Rates and Male/Female Ratios at Birth

Male/female ratio (r/(1-r))	Fertility rate (F)								
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
---------- rate of growth (%) ----------									
1.05	-1.808	-0.807	-0.091	0.468	0.927	1.316	1.655	1.955	2.224
1.10	-1.867	-0.867	-0.152	0.407	0.866	1.255	1.594	1.893	2.162
1.15	-1.925	-0.925	-0.210	0.348	0.806	1.196	1.534	1.833	2.102
1.20	-1.981	-0.982	-0.268	0.290	0.749	1.138	1.476	1.775	2.043
1.25	-2.036	-1.038	-0.324	0.234	0.692	1.081	1.419	1.718	1.986
1.30	-2.090	-1.092	-0.378	0.179	0.637	1.025	1.363	1.662	1.930
1.35	-2.142	-1.145	-0.432	0.125	0.583	0.971	1.309	1.607	1.875
1.40	-2.194	-1.197	-0.484	0.072	0.530	0.918	1.255	1.554	1.822
1.45	-2.244	-1.248	-0.536	0.021	0.478	0.866	1.203	1.501	1.769
---------- proportion of males (%) ----------									
1.05	48.4	49.1	49.6	49.9	50.1	50.2	50.4	50.4	50.5
1.10	49.5	50.3	50.7	51.0	51.2	51.4	51.5	51.6	51.7
1.15	50.6	51.3	51.8	52.1	52.3	52.5	52.6	52.7	52.8
1.20	51.6	52.3	52.8	53.1	53.3	53.5	53.6	53.7	53.8
1.25	52.6	53.3	53.8	54.1	54.3	54.5	54.6	54.7	54.8
1.30	53.5	54.3	54.7	55.0	55.3	55.4	55.6	55.7	55.8
1.35	54.4	55.2	55.6	56.0	56.2	56.3	56.5	56.6	56.7
1.40	55.3	56.0	56.5	56.8	57.0	57.2	57.4	57.5	57.5
1.45	56.1	56.8	57.3	57.6	57.9	58.1	58.2	58.3	58.4

Table 5. Effects of Higher Mortality on the Annual Rate of Growth in a Stable Alpha Population: Death Rates Increased for Female Births and Female Children

F	m/f	Percent growth when death rates increased by factor K			
		K = 1	K = 2	K = 5	K = 10
1.000	1.05	-1.808	-1.837	-1.926	-2.078
1.000	1.45	-2.244	-2.273	-2.362	-2.513
5.000	1.05	2.224	2.193	2.101	1.943
5.000	1.45	1.769	1.739	1.647	1.490

Note: See text for the basis of the calculations in this table. When K = 1 there is no change in death rates and the growth rates are the same as in Table 4.