QSEP

RESEARCH INSTITUTE FOR QUANTITATIVE
STUDIES IN ECONOMICS AND POPULATION

POPULATION CONSEQUENCES OF MALE SELECTION AT BIRTH WHEN THE SEX PROBABILITIES CAN BE ALTERED

FRANK T. DENTON
BYRON G. SPENCER

QSEP Research Report No. 452

McMaster
University
SOCIAL SCIENCES

POPULATION CONSEQUENCES OF MALE SELECTION AT BIRTH WHEN THE SEX PROBABILITIES CAN BE ALTERED

FRANK T. DENTON

BYRON G. SPENCER

QSEP Research Report No. 452

December 2013

Frank Denton and Byron Spencer are QSEP Research Associates and faculty members in the Department of Economics, McMaster University.

December, 2013

POPULATION CONSEQUENCES OF MALE SELECTION AT BIRTH WHEN THE SEX PROBABILITIES CAN BE ALTERED

Frank T. Denton and Byron G. Spencer

McMaster University

ABSTRACT

We explore the implications of male preference stopping rules for a stable population, and more generally the aggregate implications of higher male/female birth ratios. We begin by specifying nine alternative family stopping rules, derive their probability functions, and simulate the long-run effects on population growth rates and age and sex ratios. We then move away from the idea of explicit stopping rules and simulate the population effects of 81 alternative combinations of birth sex ratios and fertility rates under (implicit) preference for male children . The results show how male preference and fertility choices at the individual family level can affect the overall characteristics of a population.

JEL codes: J12, J13, C63

Keywords: Male preference; Sex selection; Stopping rules; Population effects

1. INTRODUCTION

The theoretical consequences of sex-preference stopping rules at the family level have been known for a long time – in particular the lack of any effect on the overall proportions of male and female births when the probabilities for individual births are fixed and the same throughout the population (Goodman (1961), Keyfitz (1968)). However, there is an accumulation of evidence now to indicate the use of abortion to alter those probabilities in favour of male births in countries where male preference is common; see Bongaarts (2013) for a recent survey of evidence of male preference and the use of sex-selective abortion. Some countries of Southeast Asia have received particular attention in that regard and there is evidence of the use of abortion by emigrants from those countries who are resident elsewhere: Dubuc and Coleman (2007), Almond and Edlund (2008), Abrevaya (2009), Almond et al. (2013), Ray et al. (2012). (While male preference has received most of the attention in the literature, including the present paper, female preference is certainly possible also; see Fuse (2013), for evidence of that from Japan.) Yamaguchi (1989) explored the effects of stopping rules on birth order and number of siblings in the absence of direct parental control over the sex probabilities. More recently, Yadava et al. (2013) investigated the effects on the sex ratio at birth of stopping rules when selective abortion is an option. The question on which we focus here is how changes in the sex ratio at the individual family level would translate into changes in the characteristics of the population.

We consider the implications of male preference stopping rules for a stable population, and more generally the implications of higher male/female ratios at birth. We begin by choosing a set of nine stopping rules, three with no abortion, six with, and derive the associated probability functions at the individual family level. (Our approach in this regard is similar to that of Yadava et al. 2012, although our choice of rules is somewhat different.) We simulate the consequences of each rule for the population as a whole – in particular, its rate of growth and age and sex distributions. We then move away from the idea of explicit stopping rules, specify nine alternative sex ratios at birth (which may have unspecified rules underlying them), couple the ratios with alternative fertility rates, and derive the stable population growth rates and sex distributions that result.

The instrument that we use to simulate aggregate effects is a compact Leslie matrix representing an artificial population with two sexes and broad age groups. The matrix is calibrated with realistic survival rates, allows the insertion of alternative combinations of fertility rates and sex ratios at birth, as required for particular simulations, and can be used easily to derive the resultant stable populations. We note and discuss the properties of the matrix as a prelude to its application in the simulations.

2. STOPPING RULES

We consider the following nine rules that a family might adopt. The first assumes no male preference. The next two reflect male preference but no effective way of altering the probabilities of a particular male or female birth. The remaining ones allow for the possibility of knowing the sex of a child at an early stage of pregnancy and using selective abortion to increase the probability that the next birth will be a male. In specifying the rules we abstract from miscarriages and stillbirths, and assume that in the absence of sex selective abortion a fetus would proceed to a live birth. We abstract also from the possibility of multiple births; all births are singletons. We label the stopping rules S0, S1, …, S8.

S0: There is no male preference: stop only when the number of children ever born is three.

S1: Stop when the first male child is born or when the total number of children ever born is three, whichever comes first.

S2: Stop when the second male child is born or when the number of children ever born is four, whichever comes first.

S3: Stop when the first male child is born. If there have been two births and no males, check the sex of the next fetus and abort if female. Allow the third birth to take place only if a fetus is male or there have been three successive abortions of female fetuses. The third birth will then be either male (with high probability) or female, and the three births will include one or no males.

S4: Stop when the second male child is born. If there have been three births and one or no males, check the sex of the next fetus and abort if female. Allow the fourth birth to take place if the fetus is male or there have been three successive abortions of female fetuses. The fourth birth will then be either male (with high probability) or female, and the four births will include two, one, or no males.

S5: Permit no more than one female birth; abort additional female fetuses, with no limit on the number of abortions. Stop when there are two male births.

S6: Permit no more than one female birth; abort additional female fetuses, with no limit on the number of abortions. Stop when there are three male births.

S7: Permit no more than two female births; abort additional female fetuses, with no limit on the number of abortions. Stop when there are three male births.

S8: Permit no more than two female births; abort additional female fetuses, with no limit on the number of abortions. Stop when there are four male births.

The joint probability functions for number of births (n) and number of male births (m) for these stopping rules are as follows, with p the probability of a male birth (assumed independent of parity), q = 1 – p the probability of a female birth, and *a* the probability of an abortion, which is set equal to the probability of a female fetus (*a* = q, but there is no birth); it is assumed, in the absence of abortion, that any fetus would survive to become a live birth. We put abortion functions in square brackets and place them in the probability expressions in that form to indicate their position in the sequence of births.

Rule S0: $P_0(n,m) = \binom{3}{m} p^m q^{3-m}$ (for m=0,1,2,3)

Rule S1: $P_1(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 1,2,3, m= 1)

$= q^3$ (for n = 3, m = 0)

Rule S2: $P_2(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 2,3,4, m = 2)

$= 4pq^3$ (for n = 4, m = 1)

$= q^4$ (for n = 4, m = 0)

Rule S3: $P_3(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 1,2, m = 1)

$= q^2[1+a+a^2+a^3]p$ (for n = 3, m = 1)

$= q^2[a^3]q$ (for n = 3, m = 0)

Rule S4: $P_4(n,m) = \binom{n-1}{n-m} p^m q^{n-m}$ (for n = 2,3, m = 1)

$= 4q^3[1+a+a^2+a^3]p$ (for n = 4, m = 1)

$= 3pq^2[1+a+a^2+a^3]p$ (for n = 4, m = 2)

$= q^3[a^3]q$ (for n = 4, m = 0)

Note: Rules S5 to S8 allow an unlimited number of abortions of unwanted female fetuses, and hence (in our theoretical framework) a desired male birth with certainty. In what follows, we use the symbol [1] to indicate such a male birth with probability 1, as distinguished from a natural birth with probability p.

Rule S5: $P_5(n,m) = p^2$ (for n = 2, m = 2)

$= q[1][1] + pq[1]$ (for n = 3, m = 2)

Rule S6: $P_6(n,m) = p^3$ (for n = 3, m = 3)

$= q[1][1][1] + pq[1][1] + p^2q[1]$ (for n = 4, m = 3)

Rule S7: $P_7(n,m) = p^3$ (for n = 3, m = 3)

$= 3qp^3$ (for n = 4, m = 3)

$= q^2[1][1][1] + 2pq^2[1][1] + 3p^2q^2[1]$ (for n = 5, m = 3)

Rule S8: $P_8(n,m) = p^4$ (for n = 4, m = 4)

$= 4qp^4$ (for n = 5, m = 4)

$= q^2[1][1][1][1] + 2q^2p[1][1][1] + 3q^2p^2[1][1] + 4q^2p^3[1]$ (for n = 6, m = 4)

The numerical values of these probabilities are provided in Table 1. Cells with no entries represent impossible combinations of n and m, under the specified stopping rules, and show how the rules restrict the numbers of births and the male/female combinations. Also shown in the table are E(n) and E(m) (the expected numbers of births and male births), the proportion r = E(m)/E(n), and the corresponding male/female odds ratio, m/f = E(m)/(E(n)-E(m)). The probabilities assume m/f = 1.05 for an individual birth in the absence of sex selection - a ratio common to many countries (U.S., U.K., Canada and others) - and that is also the overall ratio calculated from the expected values for stopping rules S0, S1, and S2. With a limited possibility of sex selection through abortion (maximum three times) the male/female ratio rises to 1.3 or 1.4 (stopping rules S3, S4). With unlimited use of the abortion option (rules S5 to S8) it goes as high as 3.5 (in rule S6). (The assumption of an unlimited number of abortions is a matter of theoretical convenience. A more realistic interpretation would be that the number of abortions is limited but with a maximum sufficient to drive the probability of a male birth close to 1.)

3. AN ARTIFICIAL POPULATION

We explore the effects of the various stopping rules on a population, in particular its age distribution, sex distribution, and rate of growth; beyond that, we explore, more generally, the aggregate effects of alternative combinations of fertility rates and male/female birth distributions. For those purposes we specify a closed artificial population, one that is realistic in general form but simple enough to allow easy experimentation. To emphasize its artificiality we think of it as representing a mythical country Alpha, the population dynamics of which are defined by a two-sex Leslie matrix Q of dimension 10 x10. There are five broad age groups recognized in Alpha, each of which can be viewed as consisting of 20 individual ages. For convenience we label the groups as follows: Children (ages 1 to 19), Young Adults (ages 20 to 39), Middle Aged (ages 40 to 59), Retired (60 to 79), and Old (ages 80 to 99), with no survivors at age 100. The Young Adult group is the fertile group in Alpha; women in that group bear all the children. (This simplification is convenient since it avoids having to deal with the age distribution of fertility rates, which is of little relevance for present purposes.) The population of Alpha provides a laboratory in which to ask what the aggregate effects would be of childbearing decisions made at the individual family level.

The first five rows of the Q matrix are for females, the last five for males. The Q(1,2) cell is calculated, for female babies, as s_{f0}(1-r)F, where F is the fertility rate for Young Adult females (the total rate, since there is only the one childbearing age group), r is the proportion of males at birth, and s_{f0} is the survivor correction for female births (Keyfitz and Caswell (2005)). Correspondingly, for male babies, the Q(6,2) cell is calculated as s_{m0}rF. The values of r and F are set experimentally, at various levels; s_{f0} and s_{m0} are parameters with fixed values.

The group-to-group survival rates for females are in the normal positions for a Leslie matrix: s_{f1} to s_{f4} in cells Q(2,1), Q(3,2), Q(4,3), Q(5,4). The survival rates for males are similarly in the normal positions: s_{m1} to s_{m4} in cells Q(7,6), Q(8,7), Q(9,8), Q(10,9). The survival rates are calculated from life table L_x values. (A curious feature of Alpha is that its life tables are exactly the same as the 2001 Canadian life tables, thus allowing its L_x values to be derived from that source. The Canadian tables are based on data for the three years 2000, 2001, and 2002 but are commonly referred to as 2001 tables; see Statistics Canada (2006).) All remaining cells of the Q matrix have zero values.

Each age group in Alpha consists of 20 years; correspondingly, the time interval can be thought of as 20 years, and referred to as a *generation*. Now let X_0 be a column vector representing the population at time 0. With Q fixed the population k generations later is given by $X_k = Q^k X_0$. For an arbitrary initial specification of X_0 the population can be converted to stable form (to any desired accuracy) by letting k increase until there is no further change in the proportionate age distribution. This provides a convenient procedure for simulating the effects of different specifications of F and r. (The Perron-Frobenius theorem, as adapted to a Leslie matrix, ensures ergodicity so the choice of an initial vector does not matter, as long as the elements 1 and 2 are positive; see Cull and Vogt (1973), Cohen (1979).)

4. PROPERTIES AND IMPLICATIONS OF THE Q MATRIX

The Q matrix has the following feature: the stable population form that it generates is cyclical. The matrix is imprimative, with index of imprimativity 2, and has exactly two real nonzero eigenvalues, equal in value but of opposite sign (Keyfitz and Caswell (2005)). The cycle is two generations in length so that if stability is achieved the proportionate age distributions for X_k and X_{k+2} are identical, and similarly for $X_{k+1}$and X_{k+3}. Only if the stable population is also stationary does the cycle disappear, making both the sizes and the proportionate age distributions at k and k+1 the same (the nonzero eigenvalues are then equal to 1 and -1). In the general case, to put it differently, the stable form of the population encompasses, in a 2-period sequence, both a birth effect and the subsequent echo effect resulting from the entry of last generation's newborn children into the childbearing Young Adult group this generation. While the proportionate age distributions behave cyclically the average of consecutive age distributions over the period of the cycle is strictly stable (Cull and Vogt (1973)), and that is what we use in analysing stable population characteristics. The same is true of sex distributions.

The cyclical characteristic has implications for the calculation of growth rates also. Let X_t be the population vector in the (cyclically) stable growth state at time t. We then have $X_{t+2} = Q^2X_t = (1+G)X_t$ and $X_{t+3} = Q^2X_{t+1} = (1+G)X_{t+1}$, where G is the two-generation rate of growth. The average one-generation (20-year) growth rate is then $(1+G)^{1/2} - 1$ and the average annual rate is $g = (1+G)^{1/40} - 1$. As it is common to think of population growth in terms of annual rates we show the g values in reporting aggregate results below.

The link between the growth rate and the fertility rate and proportion of males at birth can be established as follows. Let the elements of X_t be labeled X_{ti}, i = 1,…,10. Based on $X_{t+2} = Q^2X_t = (1+G)X_t$, the first element of X_{t+2} (Children) is then $X_{t+2,1} = s_{f1}s_{f0}(1\text{-}r)FX_{t1} = (1+G)X_{t1}$, and hence g can be calculated as $(s_{f1}s_{f2}(1\text{-}r)F)^{1/40} - 1$.

A check on the use of the 10x10 Leslie matrix (with only one fertile age group) to approximate the dynamics of a full, single-age life table population, is the following. With an m/f ratio of 1.05, the fertility rate required to generate a stationary population based on our Leslie matrix is virtually identical to the total fertility rate required to do the same using the full set of Canadian single-age life table death probabilities – just under 2.1 children per woman, based on a separate calculation.

5. POPULATION EFFECTS OF THE STOPPING RULES

The effects of the nine family stopping rules on the stable population of Alpha are displayed, in summary form, in Table 2. The overall proportions of male births and the corresponding male/female ratios are shown at the top of the table, along with the fertility rates and annual population growth rates. It is assumed that the proportion of women who bear no children is 15 percent in Alpha so that a fertility rate is calculated as .85 times the corresponding E(n) value in Table 1. (The choice of .85 for our fictitious country Alpha is arbitrary. For comparison, the proportion of women 40 – 44 in the United States who had given birth was reported as .82 for 2008, .90 for 1976, based on data from the Current Survey of Population; see Livingston and Cohn (2010).) Otherwise it is assumed that all families adopt the same stopping rule. Also on display in the table are the age distributions of the population (averages over the stable two-period cycle) and the proportions of males in the five age groups.

Stopping rules S0 to S2 do not allow abortion to be used to affect the sex ratio at birth; they differ in their limits on number of children at the family level, and hence the resulting fertility rates, but have no effect on the aggregate male/female ratio at birth – a general result that is well known; the m/f ratio is 1.05 in all three cases. The age distribution of the population varies but the sex distribution of the population is (necessarily) the same in all three. Rules S3 and S4 permit restricted use of abortion, a consequence being that the m/f ratio increases to almost 1.4 and the population sex distribution shifts accordingly, in favour of males – 55 or 56 percent males for the population as a whole, compared with 49 or 50 percent for the S0 – S2 no-abortion rules. The differing family limits on number of children for S3 and S4 produce a big difference in the overall fertility rates for those two rules and there are differences also in the population age distributions. A comparison of the S3 and S1 results is of particular interest: the fertility rate is the same in the two cases, 1.467, but there is a smaller proportion of young adult women to bear children with the S3 rule, and hence a more rapid rate of decline in the population.

Stopping rules S5 to S8 permit unlimited use of abortion (or, if one prefers, limited use, but with the limits high enough that the result is essentially the same). The m/f ratio at birth for these rules ranges from about 2.0 to 3.5 - very high ratios but ones that allow the effects of male birth selection to stand out clearly in our analytical framework. These rules produce an Alpha population with an overall proportion of males of 65.0 percent at the low end, 76.1 percent at the upper end, for the population as a whole, and higher ratios still for the younger age groups in the population. Now the offsetting effect of fewer women of childbearing age is obvious: S6 produces a fertility rate of 3.3 but a population that is declining at about .8 percent per year; S5 produces a fertility rate of 2.3 (still well above the natural replacement level) but a population that is declining at 1.2 percent a year.

6. INTERACTION OF THE FERTILITY RATE AND THE SEX RATIO AT BIRTH

We now move away from the idea of specific stopping rules and consider simply the implications of alternative combinations of fertility rates and male/female ratios at birth. (One may think of there being sex selection and family size preferences underlying the combinations but they are no longer explicit.) We focus in particular on the growth rate and the overall proportion of males in the population of our fictitious country Alpha, choosing nine alternative fertility rates, from 1.0 to 5.0, and coupling them with nine alternative m/f ratios, from 1.05 to 1.45. The results are presented in Table 3. The choice of fertility rates reflects in a rough way the range of rates observed among countries in recent decades. The m/f ratios are a convenient choice for exploring the effects of variations in those ratios: the lower bound, 1.05, is a commonly observed ratio; the upper bound, 1.45, is an arbitrarily high value chosen for exploratory purposes.

Results for the 81 combinations of fertility rates and m/f ratios are shown in Table 3. The effect of increasing the sex ratio at birth in favour of males is to reduce the rate of growth and increase the proportion of males (decrease the proportion of females) in the population, for any given fertility rate. With a fertility rate of 2.5 children per woman and an m/f ratio of 1.05 the population grows at a rate of .468 percent per year; if the m/f ratio is increased to 1.3, the growth rate falls to .179 percent, and if the ratio is allowed to increase further, to 1.45, the rate falls to only .021 percent, almost no growth at all. To put it differently, the natural replacement fertility rate – the rate required to produce a stationary population - is about 2.1 children per woman with the empirically common m/f ratio 1.05; with a very high ratio of 1.45 the natural replacement rate is about 2.5 children per woman. Very roughly (a “rule of thumb" observation), for any given fertility rate, an increase of .05 in m/f lowers the rate of stable population growth by .05 to .06 percent per year as a consequence of the reduced proportion of females in the fertile Young Adult age group.

The decline in the proportion of males in the overall population ranges from 48.4 percent at one extreme (F = 1.0, m/f = 1.05) to 58.4 percent at the other (F = 5.0, m/f = 1.45). Another rough ("rule of thumb") observation, similar to the one in the previous paragraph, is that, given any fertility rate, an increase of .05 in m/f produces an increase of .8 to 1.1 in the percentage of males.

7. CONCLUSION

Alpha is a mythical country and the results of our simulations are theoretical. Underlying our presentation of the results is the idea that abortion can be used to alter the male/female distribution of births. There is much evidence to indicate that that is indeed the case among some families in some parts of the world – some countries of Southeast Asia, in particular, and among some emigrants from those countries. But what has been observed – or rather inferred from census or birth registration data - is a far cry from the use of abortion on a scale wide enough to have the kinds of effects that our simulations generate at the population level. We think the results are of legitimate interest from a theoretical point of view but they are just that, theoretical.

We are accustomed to thinking of the sex ratio at birth as fixed for population projection purposes – virtually a parameter, in effect. One must of course project fertility and mortality rates but the value of the sex ratio is unchanging and can be taken as given. However that need not always be the case. There is evidence of systematic variation in the ratio over time in some countries and it may be necessary to allow for that in making projections; see for example the projections for China and India by Guilmoto (2010).

The use of abortion as an instrument to implement male preference has been publicly condemned and in some countries prohibited, though with limited effect. Thinking about sex selection more broadly, could results such as those presented in this paper have a more realistic interpretation in the future? Technological development has provided highly effective methods of birth control that are in widespread use today, and which have had profound population implications. Could there be similar changes in the future that would make parental sex selection without abortion a realistic voluntary possibility on a large scale, with attendant effects on the population? There are procedures today that offer the possibility of sex selection (beyond the use of abortion) and their availability is advertised commercially (go to Google and type in "sex selection at birth"). But they are far from being widespread and inexpensively applicable. Can we expect that to change; will the population effects of sex selection take on more practical significance? We close this paper with the following suggestion by Keyfitz and Caswell (2005, p. 438) on what might happen if male preference were actually to bring about a significant shift in the sex ratio at birth: "Within a decade or so of the birth of a disproportionate number of boys couples would come to value girls more highly. Perhaps a series of waves would ensue, not unlike those familiar in a market economy … ." An interesting speculation.

REFERENCES

Abrevaya, J. (2009). Are there missing girls in the United States? Evidence from birth data. *American Economic Journal: Applied Economics*, *1*, 1-34.

Almond, D., & Edlund, L. (2008). Son-biased sex ratios in the 2000 United States census. *Proceedings of the National Academy of the United States of America*, *105*, 5681-5682.

Almond, D., Edlund, L., & Milligan, K. (2013). Son preference and the persistence of culture: Evidence from South and East Asian immigrants to Canada. *Population and Development Review, 39*, 75-95.

Bongaarts, J. (2013). The implementation of preferences for male offspring. *Population and Development Review*, *39*, 185-369.

Cohen, J. E. (1979). Ergodic theorems in demography. *Bulletin of the American Mathematical Society (New Series), 1*, 275-293.

Cull, P., & Vogt, A. (1973). Mathematical analysis of the asymptotic behavior of the Leslie matrix model. *Bulletin of Mathematical Biology*, *35*, 645-661.

Dubuc, S., & Coleman, D. (2007). An increase in the sex ratio of births to India-born mothers in England and Wales: Evidence for sex-selective abortion. *Population and Development Review*, *33*, 383-400.

Fuse, K. (2013). Daughter preference in Japan: A reflection of gender role attitudes? *Demographic Research, 28*, 1021-1052.

Goodman, L. A. (1961). Some possible effects of birth control on the human sex ratio. *Annals of Human Genetics, 25*, 75-81.

Guilmato, C. Z. (2010). Longer-term disruptions to demographic structures in China and India resulting from skewed sex ratios at birth. *Asian Population Studies, 6*, 3-24.

Keyfitz, N. (1968). *Introduction to the Mathematics of Population*. Addison-Wesley Publishing Company Inc.

Keyfitz, N., & Caswell, H. (2005). *Applied Mathematical Demography.* (3rd ed.) Springer Science+Business Media Inc.

Livingston, G., & Cohn, D. (2010). Childlessness up among all women; down among women with advanced degrees. *Pew Research Center: Social and Demographic Trends*, July 25, 2010.

Ray, J. G., Henry, D. A., & Urquia, M. L. (2012). Sex ratios among Canadian live born infants of mothers from different countries. *Canadian Medical Association Journal*, *184*, E492-E496.

Statistics Canada (2006). *Life tables, Canada, provinces and territories 2000 to 2002*. Ottawa. Catalogue 84-537-XIE.

Yadava, R. C., Kumar, A., & Srivastava, U. (2013). Sex ratio at birth: A model based approach. *Mathematical Social Sciences, 65*, 36-39.

Yamaguchi, K. (1989). A formal theory for male-preferring stopping rules of childbearing sex differences in birth order and in the number of siblings. *Demography, 26*, 451-465.

Table 1. Probability Distributions and Related Statistics at the Family Level: Numbers of Births and Numbers of Male Births Under Alternative Stopping Rules

Number of births (n)	Number of males (m)	Prob(n,m) under given stopping rule								
		S0	S1	S2	S3	S4	S5	S6	S7	S8
1	0	--	--	--	--	--	--	--	--	--
1	1	--	.512	--	.512	--	--	--	--	--
2	0	--	--	--	--	--	--	--	--	--
2	1	--	.250	--	.250	--	--	--	--	--
2	2	--	--	.262	--	.262	.262	--	--	--
3	0	.116	.116	--	.013	--	--	--	--	--
3	1	.366	.122	--	.224	--	--	--	--	--
3	2	.384	--	.256	--	.256	.738	--	--	--
3	3	.134	--	--	--	--	--	.134	.134	--
4	0	--	--	.057	--	.007	--	--	--	--
4	1	--	--	.238	--	.130	--	--	--	--
4	2	--	--	.187	--	.345	--	--	--	--
4	3	--	--	--	--	--	--	.866	.197	--
4	4	--	--	--	--	--	--	--	--	.069
5	0	--	--	--	--	--	--	--	--	--
5	1	--	--	--	--	--	--	--	--	--
5	2	--	--	--	--	--	--	--	--	--
5	3	--	--	--	--	--	--	--	.669	--
5	4	--	--	--	--	--	--	--	--	.134
5	5	--	--	--	--	--	--	--	--	--
6	0	--	--	--	--	--	--	--	--	--
6	1	--	--	--	--	--	--	--	--	--
6	2	--	--	--	--	--	--	--	--	--
6	3	--	--	--	--	--	--	--	--	--
6	4	--	--	--	--	--	--	--	--	.797
	E(n)	3.000	1.726	3.219	1.726	3.219	2.738	3.866	4.535	5.728
	E(m)	1.537	.884	1.649	.987	1.857	2.000	3.000	3.000	4.000
	E(m)/E(n)	.512	.512	.512	.572	.577	.731	.776	.662	.698
	m/f ratio	1.050	1.050	1.050	1.335	1.362	2.711	3.466	1.955	2.315

Note: In the absence of selective abortion a birth is assumed to be male with probability .5122. A double dash indicates an impossible n,m combination.

Table 2. Stable Alpha Populations Resulting from Alternative Stopping Rules

	Stopping rule								
	S0	S1	S2	S3	S4	S5	S6	S7	S8
Proportion male births (r)	.512	.512	.512	.572	.577	.731	.776	.662	.698
Male/female ratio at birth	1.050	1.050	1.050	1.335	1.362	2.711	3.466	1.955	2.315
Fertility rate (F)	2.550	1.467	2.736	1.467	2.736	2.327	3.286	3.855	4.869
Annual % growth rate	.517	-.862	.695	-1.184	.339	-1.189	-.793	.637	.936
Population age distribution									
- children	29.2	18.6	30.7	16.5	27.8	16.7	19.5	30.4	32.9
- young adults	26.1	21.9	26.5	20.8	25.8	21.1	22.6	26.5	27.1
- middle aged	22.8	25.3	22.3	25.5	23.3	25.8	25.6	22.6	21.7
- retired	17.0	24.8	16.0	26.6	17.8	26.5	24.1	16.2	14.6
- old	4.9	9.4	4.5	10.6	5.2	9.9	8.1	4.3	3.6
- all ages	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Proportion males									
- children	51.2	51.2	51.2	57.1	57.6	73.0	77.6	66.1	69.8
- young adults	51.0	51.0	51.0	57.0	57.5	72.9	77.5	66.0	69.7
- middle aged	50.6	50.6	50.6	56.6	57.1	72.6	77.2	65.6	69.3
- retired	48.2	48.2	48.2	54.2	54.7	70.6	75.5	63.4	67.3
- old	38.7	38.7	38.7	44.5	45.0	62.0	67.6	54.1	58.2
- all ages	49.9	49.1	50.0	54.9	56.3	71.2	76.1	65.0	68.9

Note: The proportion of women bearing no children is set at 15% for this table.

Table 3. Annual Rate of Growth and Proportion of Males in a Stable Alpha Population with Alternative Combinations of Fertility Rates and Male/Female Ratios at Birth

	Fertility rate (F)								
Male/female ratio (r/(1-r))	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
	---------- rate of growth (%) ----------								
1.05	-1.808	-.807	-.091	.468	.927	1.316	1.655	1.955	2.224
1.10	-1.867	-.867	-.152	.407	.866	1.255	1.594	1.893	2.162
1.15	-1.925	-.925	-.210	.348	.806	1.196	1.534	1.833	2.102
1.20	-1.981	-.982	-.268	.290	.749	1.138	1.476	1.775	2.043
1.25	-2.036	-1.038	-.324	.234	.692	1.081	1.419	1.718	1.986
1.30	-2.090	-1.092	-.378	.179	.637	1.025	1.363	1.662	1.930
1.35	-2.142	-1.145	-.432	.125	.583	.971	1.309	1.607	1.875
1.40	-2.194	-1.197	-.484	.072	.530	.918	1.255	1.554	1.822
1.45	-2.244	-1.248	-.536	.021	.478	.866	1.203	1.501	1.769
	---------- proportion of males (%) ----------								
1.05	48.4	49.1	49.6	49.9	50.1	50.2	50.4	50.4	50.5
1.10	49.5	50.3	50.7	51.0	51.2	51.4	51.5	51.6	51.7
1.15	50.6	51.3	51.8	52.1	52.3	52.5	52.6	52.7	52.8
1.20	51.6	52.3	52.8	53.1	53.3	53.5	53.6	53.7	53.8
1.25	52.6	53.3	53.8	54.1	54.3	54.5	54.6	54.7	54.8
1.30	53.5	54.3	54.7	55.0	55.3	55.4	55.6	55.7	55.8
1.35	54.4	55.2	55.6	56.0	56.2	56.3	56.5	56.6	56.7
1.40	55.3	56.0	56.5	56.8	57.0	57.2	57.4	57.5	57.5
1.45	56.1	56.8	57.3	57.6	57.9	58.1	58.2	58.3	58.4