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Aggregation and Other Biases in the Calculation of Consumer Elasticities for Models of Arbitrary Rank

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Consumer-related policy decisions often require analysis of aggregate responses or mean elasticities. However, in practice these mean elasticities are seldom used. Mean elasticities can be approximated using aggregate data, but that introduces aggregation bias for full and compensated price elasticities, though interestingly not for expenditure elasticities. The biases corresponding to incorrect approximations of mean elasticities depend on the type of data (micro or aggregate), the type and rank of the model, and generalized measures of income inequality. These biases are distinct from the biases (already noted in the literature) when using aggregate data to estimate micro elasticites at mean income.

Keywords: Aggregate price and expenditure elasticities, aggregation bias, consumer demand, generalized measures of income inequality, income distribution

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of Consumer Elasticities for Models of Arbitrary Rank

1. Introduction

Economic policy decisions frequently require the evaluation of aggregate responses - the aggregate response of consumer expenditure on gasoline to a gasoline tax, say, or the aggregate effect of an income supplement on the demand for rental housing.[1] Such responses are often represented best in the form of aggregate elasticities - or mean elasticities, since the two are the same. True mean elasticity formulas are seldom used, though. Elasticities are often reported at mean or other income levels for models fitted to micro data and elasticities calculated from aggregate data (and hence subject to aggregation bias) are often interpreted as elasticities at mean income. But elasticities at the mean are not the same as mean elasticities; they are approximations at best and fail to take into account the characteristics of the income distribution. We derive in this paper exact formulas for mean price and expenditure elasticities for models of arbitrary rank and arbitrary income distribution. We derive also formulas for the biases resulting from the use of elasticities at the mean to represent mean elasticities and from the use of aggregate rather than micro data in the calculation of either. We then show what the biases look like for four familiar consumer expenditure models. (The biases are theoretical. They are illustrated numerically but the paper is not concerned with issues of statistical estimation.)

Three types of elasticities are of interest: expenditure elasticities, full (uncompensated) price elasticities, and compensated price elasticities. (Full price elasticities may be of practical importance for policy forecasting - forecasting the revenue yield of the gasoline tax, for example - while compensated price elasticities are of more interest from a welfare point of view.) We consider three situations (in describing them

[1] In their survey on how to account for heterogeneity in aggregation, Blundell and Stoker (2005) begin their discussion by emphasizing that "some of the most important questions in economics…concern economic aggregates." Economics "is often concerned with…aggregate consumption and savings, market demand and supply, total tax revenues,… and so forth." Moreover, Slottje (2008) points to the recent "experiment of the US government in pumping over $50 billion dollars into consumers' hands to jump start the US economy in 2008" as an exemplification of "the importance of understanding aggregate consumer behavior and what does and does not impact it."

we follow the frequent practice in the literature of using "income" as equivalent to total expenditure, in references to the income distribution):

(1) Micro data are available and are used to calculate mean (or aggregate) elasticities.

(2) Micro data are available and are used to calculate elasticities at the mean of the income distribution. The elasticities at the mean are then used as approximations to mean elasticities.

(3) Only aggregate data are available (time series, say) and those are used to estimate the underlying micro model and corresponding elasticities. The elasticities are interpreted as if they were mean or "representative consumer" elasticities in the micro model, and possibly used to represent the aggregate effects of a price or income change.[2]

We derive the formulas for calculating the mean elasticities in situation (1) and the biases implicit in situations (2) and (3). The biases depend on the structure of the income distribution, irrespective of whether micro data or aggregate data are used. But there is an interesting exception: calculations of mean expenditure elasticities based on aggregate data are unbiased; regardless of the income distribution there is no aggregation error.

2. Framework

Assume I commodities, indexed by i, K households, indexed by k, and a common price vector $p = (p_1, p_2, ..., p_I)$ (sometimes referred to as the law of one price). Household k spends x_{ik} units of income to purchase $q_{ik} > 0$ units of commodity i, has

[2]In spite of the increased availability and obvious advantages of micro data sets it is still the case that aggregate data must often be used in estimating consumer demand models. Of 21 published articles surveyed by the present authors, 15 used aggregate data in the estimation of "almost ideal demand systems," either AIDS or QUAIDS (Denton and Mountain, 2007). The reasons no doubt vary: lack of availability of micro data for a particular country or region, lack of sufficient commodity detail required for a particular purpose, or of observations on particular explanatory variables, the need to use time series available only at the aggregate level in order to estimate a model with dynamic properties, and so on. We note too that much of the attention given to elasticities calculated from aggregate data in the literature has focused on their use as estimates of underlying micro elasticities, much less on their use as estimates of aggregate elasticities, even though the latter are often of greater policy relevance.

total income (expenditure) x_k, and thus an expenditure share $w_{ik} = \frac{x_{ik}}{x_k}$. Now consider, for some arbitrary R, the generic expenditure system

$$w_{ik} = \sum_{r=0}^{R-1} \frac{\tilde{c}_{ri}(p)}{x_k} f_r(x_k, p) \quad (i = 1,2,...,I) \tag{1}$$

where $f_r(x_k, p) = f_r\left(\frac{x_k - d(p)}{b(p)}\right)$ for a translated and deflated system, the $\tilde{c}_{ri}(p)$ can be interpreted as coefficients, conditional on p, and the functions $d(p)$ and $b(p)$ are homogeneous of degree one.[3] (Note that demographic, geographic, and other such household characteristics commonly included as additional variables in expenditure models can be accommodated in $\tilde{c}_{0i}$(p) and $d(p)$.) The rank of the demand system is the maximum number of dimensions spanned by the system's Engle curves. Equation (1) nests Gorman's (1981) rank 3 rationally derived system, Lewbel's (1989a) rank 4 rationally derived system, and Lewbel's (2003) translated deflated income system. At the level of specific applicable models it nests such well known ones as the translog (Christensen, Jorgenson, and Lau, 1975), AIDS (Deaton and Muellbauer, 1980), and QUAIDS (Banks, Blundell, and Lewbel, 1997). More generally, it is consistent with many studies in which expenditure systems have been found to be well approximated by finite (invariably low) order log-income polynomials. In the case of rank 2 and rank 3 polynomials in logarithms of deflated expenditures, such as translog, AIDS and QUAIDS, equation (1) simplifies to

$$w_{ik} = \sum_{r=0}^{R-1} a_{ri}(p)(\ln x_k)^r \quad (i = 1,2,...,I) \qquad \text{for } R = 2,3 \tag{2}$$

[3] To obtain this expenditure system, we could begin with

$$q_{ik} = \sum_{r=0}^{R-1} \tilde{\tilde{c}}_{ri}(p) f_r(x_k, p) \quad (i = 1,2,...,I) \quad where\ \tilde{\tilde{c}}_{ri}(p) = \frac{\tilde{c}_{ri}(p)}{p_i}$$

by dropping the translation term $d(p)$ and by setting $f_r(x_k, p) = \frac{x_k}{b(p)}\left[\ln\left(\frac{x_k}{b(p}\right)\right]^r$

and $a_{ri}(p) = b(p)\sum_{j=r}^{R-1}\tilde{c}_{ji}(p)\binom{j}{j-r}(-\ln(b(p))^{j-r}$. Here R is the rank of the system.

Reformulating $\frac{f_r(x_k, p)}{x_k}$ in equation (1) as a Taylor series expansion in $\ln x_k$ around $\ln x_k = 0$ ($x_k = 1$), and using the notation f_r to denote the function $f_r(x_k, p)$, results in

$$w_{ik} = \sum_{r=0}^{R-1}\tilde{c}_{ri}(p)\sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{\partial^m f_r}{\partial \ln x_k{}^m} + (-1)^m f_r\right)\Big|_{x_k=1}(\ln x_k)^m \quad (i = 1,2,..., I),$$

with $\frac{\partial^0 f_0}{\partial \ln x_k{}^0} = f_0$.

With further regrouping of terms involving $(\ln x_k)^m$, this can be further simplified to

$$w_{ik} = \sum_{m=0}^{\infty} c_{mi}(p)(\ln x_k)^m \quad (i = 1,2,..., I) \qquad (3)$$

where $c_{mi}(p) = \sum_{r=0}^{R-1}\tilde{c}_{ri}(p)\frac{1}{m!}\left(\frac{\partial^m f_r}{\partial \ln x_k{}^m} + (-1)^m f_r\right)\Big|_{x_k=1}$

The fact that equation (3) nests equation (2) can be seen by setting to zero all the derivatives of order higher than 2.

Elasticities (the focus of this paper) are invariant to scalar transformations of the units of measurement for income and prices. This allows us to simplify notation, without loss of generality (and with no implications for how a model might actually be estimated in practice), by introducing the normalization restrictions $p_i = 1, \forall i$, and $\bar{x} = 1$ where $\bar{x} = \sum_{k=1}^{K} \frac{x_k}{K}$. Hereafter we write simply c_{mi}, if the context permits.

We now need an appropriate way of characterizing the income distribution. To that end we write $X = \sum_{k=1}^{K} x_k$, $y_k = \frac{x_k}{X}$, and $h_m = \sum_{k=1}^{K} y_k (\ln x_k - \ln \bar{x})^m$ $(m = 0,1,2,...)$. We can interpret h_m as a generalized measure of inequality (GMI) of order m. This is a straightforward mathematical generalization of Theil's (1967) measure of inequality, which is obtained by setting $m = 1$, and which was inspired by Shannon's (1948) measure of information entropy. An arbitrary income distribution can then be characterized by the sequence $h_0, h_1, h_2,$ etc. (Note that $h_0 = 1$. Note too that $h_m = 0$ for all $m > 0$ when the distribution is uniform.) Invoking the normalization restriction $\bar{x} = 1$ allows the simpler definition $h_m = \sum_{k=1}^{K} y_k (\ln x_k)^m$.

The GMIs provide a bridge from the micro specification of equation (3) to the corresponding specification at the aggregate level. Let X_i be aggregate expenditure on commodity i by all households and let $W_i = \frac{X_i}{X} = \sum_{k=1}^{K} w_{ik} y_k$ be the aggregate expenditure share. Then

$$W_i = \sum_{m=0}^{\infty} c_{mi} \sum_{k=1}^{K} y_k (\ln x_k)^m = \sum_{m=0}^{\infty} c_{mi} h_m \qquad (4)$$

For polynomials in logarithms of deflated expenditures defined in equation (2) for $R = 2,3$, the aggregate expenditure share is

$$W_i = \sum_{r=0}^{R-1} a_{ri} \sum_{k=1}^{K} y_k (\ln x_k)^r = \sum_{r=0}^{R-1} a_{ri} h_r \qquad (5)$$

Here, the W_i depend on GMIs up to order $R-1$ and the GMIs of order R and higher, which may be required to fully characterize some arbitrarily specified income distribution, are irrelevant for the determination of W_i. However, in this case GMIs up to order $2(R-1)$ are required for the determination of some elasticities and corresponding biases, as we show below.

3. Mean Elasticities

Household k has a full (uncompensated) elasticity of demand for commodity i with respect to the price of commodity j, $\frac{\partial \ln q_{ik}}{\partial \ln p_j}$, and a compensated elasticity

$$\left.\frac{\partial \ln q_{ik}}{\partial \ln p_j}\right|_{\bar{U}} = \frac{\partial \ln q_{ik}}{\partial \ln p_j} + w_{jk}\frac{\partial \ln q_{ik}}{\partial \ln x_k} \tag{6}$$

where $\bar{U}$ indicates the constancy of utility. Now write $Q_i = \sum_{k=1}^{K} q_{ik}$ for aggregate purchases of commodity i, all households combined, ϕ_{1ij} for the mean (same as aggregate) full price elasticity, and η_{1ij} for the mean compensated price elasticity. (The significance of the 1 subscript will be apparent later.) We then have

$$\phi_{1ij} = \frac{\partial \ln Q_i}{\partial \ln p_j} = \sum_{k=1}^{K}\frac{\partial \ln q_{ik}}{\partial \ln p_j}\cdot\frac{q_{ik}}{Q_i}$$

$$\eta_{1ij} = \phi_{1ij} + \sum_{k=1}^{K} w_{jk}\frac{\partial \ln q_{ik}}{\partial \ln x_k}\cdot\frac{q_{ik}}{Q_i} \tag{7}$$

where it is assumed (in the derivation of η_{1ij}) that households have a common utility function (but may of course be at different points on that function).

The expenditure elasticity for commodity i and for household k is $\frac{\partial \ln q_{ik}}{\partial \ln x_k}$. To derive a corresponding mean elasticity it is necessary to stipulate how a proportional increase in aggregate income is shared among households. The most straightforward assumption, and the one that we make, is that the proportional change is the same for all households, so that $\frac{\partial \ln x_k}{\partial \ln X} = 1$ for all k.[4] Writing ε_{1i} for the mean expenditure elasticity we then have

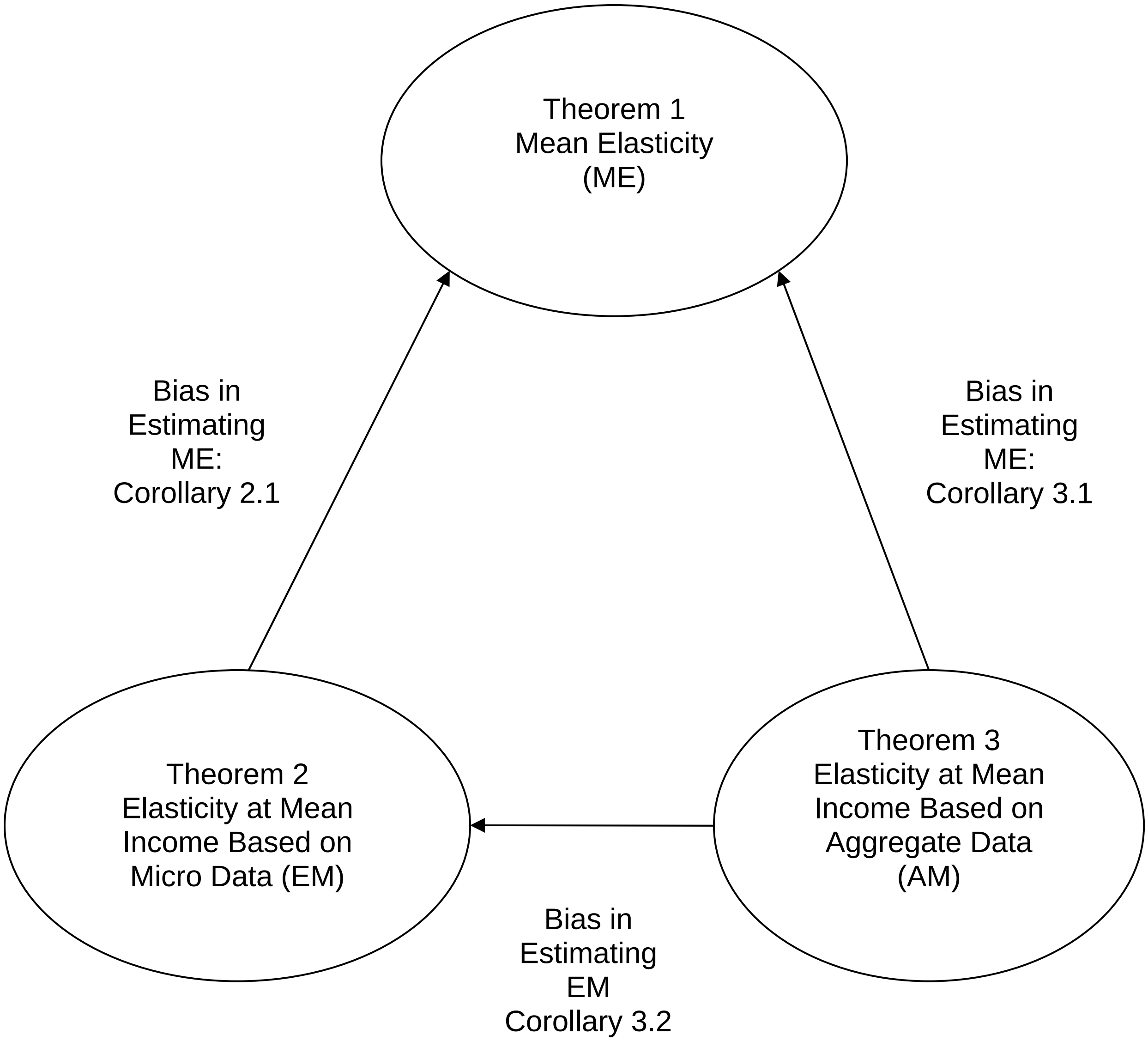
$$\varepsilon_{1i} = \frac{\partial Q_i}{\partial X} \cdot \frac{X}{Q_i} = \sum_{k=1}^{K} \frac{\partial q_{ik}}{\partial x_k} \cdot \frac{x_k}{q_{ik}} \cdot \frac{q_{ik}}{Q_i} \cdot \frac{X}{x_k} \cdot \frac{\partial x_k}{\partial X} = \sum_{k=1}^{K} \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i} \qquad (8)$$

4. Calculations with Micro Data

Given an appropriate set of data for individual households and an expenditure system defined by equation (3), price and expenditure elasticities can be calculated directly, whatever the distribution of income. These elasticities are the correct ones for evaluating aggregate effects. Elasticities at the mean of the income distribution can also be calculated, either for their own value or as (biased) approximations to the mean elasticities. We present the results of these calculations in the form of two theorems and a corollary. (All proofs are provided in Appendix A, both for this section and the next.) Figure 1 provides a schematic illustration of the relationship among these two theorems (theorems dealing with the use of micro data to calculate the elasticity at the mean of the income distribution (EM), and the mean elasticity (ME)) and our third theorem, which is concerned with estimating the elasticity at mean income based on aggregate data (AM). The biases in using one of these elasticities (EM or AM) to estimate another (ME or EM), as identified in the corollaries, are correspondingly labeled in the figure.

[4] This assumption is consistent with what Lewbel (1989b, 1990) calls "mean scaling."

Figure 1: Relationships Among Elasticity Theorems and Associated Biases



Note: For rank 2 and rank 3 models, with expenditure shares expressed as polynomials in logarithms of deflated expenditures, Corollaries 2.1.1, 3.1.1 and 3.2.1 are embedded in Corollaries 2.1, 3.1 and 3.2, respectively.

There are no constraints on the rank of a demand system with regard to the existence of expenditure or full price elasticities.[5] However, the existence of compensated price elasticities (under the assumption of rationality) requires the rank to be at most four (Lewbel, 1989a). Thus while the following theorems relate to systems of arbitrary rank they have meaning for compensated price elasticities only for systems up to rank four.

Theorem 1: Calculation of mean elasticities (ME) using micro data:

(i) The mean full price elasticity is given by

$$\phi_{1ij} = -\delta_{ij} + \frac{\sum_{m=0}^{\infty}\left(\frac{\partial c_{mi}}{\partial \ln p_j}\right)h_m}{\sum_{m=0}^{\infty} c_{mi} h_m},$$

where δ_{ij} is the Kronecker delta.

(ii) The mean compensated price elasticity is

$$\eta_{1ij} = -\delta_{ij} + \frac{\sum_{m=0}^{\infty}\left(\frac{\partial c_{mi}}{\partial \ln p_j}\right)h_m + \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} m c_{mi} c_{nj} h_{m+n-1} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} c_{mi} c_{nj} h_{m+n}}{\sum_{m=0}^{\infty} c_{mi} h_m}.$$

(iii)The mean expenditure elasticity is

$$\varepsilon_{1i} = 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m}.$$

Note that these three elasticities represent the correct (unbiased) values. The biases relating to mean elasticities derived in Corollaries 2 and 3 below are thus differences from these values.

[5] Lau (1977) develops a theory of exact aggregation for systems of any rank, where aggregate demand can be expressed in terms of index functions such as the GMIs that we are using.

Theorem 2: Calculation of elasticities at mean income (EM) using micro data:

(i) The full price elasticity at mean income is

$$\phi_{2ij} = -\delta_{ij} + \frac{\left[\dfrac{\partial c_{0i}}{\partial \ln p_j}\right]}{c_{0i}} \quad .$$

(ii) The compensated price elasticity at mean income is

$$\eta_{2ij} = -\delta_{ij} + \frac{\left[\dfrac{\partial c_{0i}}{\partial \ln p_j}\right] + c_{oj} c_{1i}}{c_{0i}} + c_{0j}.$$

(iii) The expenditure elasticity at mean income is

$$\varepsilon_{2i} = 1 + \frac{c_{1i}}{c_{0i}}.$$

Note that all three elasticities are independent of the income distribution.

Corollary 2.1: Biases in interpreting elasticities at mean income (EM) to represent mean elasticities (ME):

(i) The bias for the full price elasticity is

$$\phi_{2ij} - \phi_{1ij} = \frac{\left(\dfrac{\partial c_{0i}}{\partial \ln p_j}\right) \sum_{m=0}^{\infty} c_{mi} h_m - \left(\sum_{m=0}^{\infty} \left(\dfrac{\partial c_{mi}}{\partial \ln p_j} \right) h_m \right) c_{0i}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad .$$

(ii) The bias for the compensated price elasticity is

$$\eta_{2ij}-\eta_{1ij}=\frac{\left\{\begin{array}{l}\left(\left(\left(\dfrac{\partial c_{0i}}{\partial \ln p_j}+c_{0j}c_{0i}\right)+c_{oj}c_{1i}\right)\sum_{m=0}^{\infty}c_{mi}h_m-\right.\\ \left.\left(\sum_{m=0}^{\infty}\left(\dfrac{\partial c_{mi}}{\partial \ln p_j}\right)h_m+\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}mc_{mi}c_{nj}h_{m+n-1}+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}c_{mi}c_{nj}h_{m+n}\right)c_{0i}\right)\end{array}\right\}}{c_{0i}\sum_{m=0}^{\infty}c_{mi}h_m}\quad .$$

(iii) The bias for the expenditure elasticity is

$$\varepsilon_{2i}-\varepsilon_{1i}=\frac{c_{1i}\sum_{m=0}^{\infty}c_{mi}h_m-c_{0i}\sum_{m=1}^{\infty}mc_{mi}h_{m-1}}{c_{0i}\sum_{m=0}^{\infty}c_{mi}h_m}\quad .$$

Corollary 2.1.1: For rank 2 and 3 $(R=2,3)$ polynomials in logarithms of deflated expenditures defined in equation (2), biases in interpreting elasticities at mean income as mean elasticities:

(i) The bias for the full price elasticity is

$$\phi_{2ij}-\phi_{1ij}=\frac{\left(\dfrac{\partial a_{0i}}{\partial \ln p_j}\right)\sum_{r=0}^{R-1}a_{ri}h_r-\left(\sum_{r=0}^{R-1}\left(\dfrac{\partial a_{ri}}{\partial \ln p_j}\right)h_r\right)a_{0i}}{a_{0i}\sum_{r=0}^{R-1}a_{ri}h_r}$$

and thus is a function of GMIs up to order $R-1$.

(ii) The bias for the compensated price elasticity is

$$\eta_{2ij}-\eta_{1ij}=\frac{\left\{\begin{array}{l}\left(\left(\left(\dfrac{\partial a_{0i}}{\partial \ln p_j}+a_{0j}a_{0i}\right)+a_{oj}a_{1i}\right)\displaystyle\sum_{r=0}^{R-1}a_{ri}h_r-\right.\\ \left.\left(\displaystyle\sum_{r=0}^{R-1}\left(\dfrac{\partial a_{ri}}{\partial \ln p_j}\right)h_r+\sum_{r=1}^{R-1}\sum_{s=0}^{R-1}ra_{ri}a_{sj}h_{r+s-1}+\sum_{r=0}^{R-1}\sum_{s=0}^{R-1}a_{ri}a_{sj}h_{r+s}\right)a_{0i}\right)\end{array}\right\}}{a_{0i}\displaystyle\sum_{r=0}^{R-1}a_{ri}h_r}$$ and thus is a function of GMIs up to order $2(R-1)$.

(iii) The bias for the expenditure elasticity is

$$\varepsilon_{2i}-\varepsilon_{1i}=\frac{a_{1i}\sum_{r=0}^{R-1}a_{ri}h_r-a_{0i}\sum_{r=1}^{R-1}ra_{ri}h_{r-1}}{a_{0i}\sum_{r=0}^{R-1}a_{ri}h_r}$$

and thus is a function of GMIs up to order $R-1$.

5. Calculations with Aggregate Data

Micro data are often not available, or not suitable, for the estimation of particular models and elasticities, and aggregate data may have to be used (see footnote 2), thus introducing the possibility of aggregation bias. The common practice is to assume that the micro model holds at the aggregate level, which in general it does not – to assume, that is, that equation (3) holds with w_{ik} and x_k replaced by their aggregate counterparts. On that basis the variant of equation (3) employed when using aggregate data is

$$W_i=\sum_{m=0}^{\infty}\hat{c}_{mi}(p)\left[\ln\bar{x}\right]^m \quad (i=1,2,...,I) \qquad (9)$$

where $\hat{c}_{mi}(p)=\sum_{n=m}^{\infty}e_{mn}c_{ni}(p)h_{n-m}$, $e_{0m}=e_{mm}=1$ for $m=0,1,2,...$,

$e_{mn}=e_{m,n-1}+e_{m-1,n-1}$ for $m=1,2,...,$ $n=m+1,m+2,...$.

The associated full price, compensated price, and expenditure elasticities for this model are obtained by calculating

$$\phi_{3ij} = \frac{\partial \ln Q_i}{\partial \ln p_j}, \quad \eta_{3ij} = \frac{\partial \ln Q_i}{\partial \ln p_j}\Big|_{\bar{U}}, \quad \text{and } \varepsilon_{3i} = \frac{\partial \ln Q_i}{\partial \ln X},$$

where

$$\eta_{3ij} = \phi_{3ij} + W_j \varepsilon_{3i} . \qquad (10).$$

The formulas for these calculations using aggregate data are stated in the following theorem:

Theorem 3: Calculation of elasticities at mean income using aggregate data (AM), based on equation (9):

(i) The full price elasticity is

$$\phi_{3ij} = -\delta_{ij} + \frac{\dfrac{\partial c_{0i}}{\partial \ln p_j} - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m} .$$

(ii) The compensated price elasticity is

$$\eta_{3ij} = -\delta_{ij} + \frac{\dfrac{\partial c_{0i}}{\partial \ln p_j} + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nj} c_{mi} h_n h_m + \sum_{m=1}^{\infty} m c_{0j} c_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} .$$

(iii) The expenditure elasticity is

$$\varepsilon_{3i} = 1 + \frac{\sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} .$$

If the formulas in Theorem 3 are applied, and the results are interpreted as true mean elasticities, the biases are as given in Corollary 3.1. If on the other hand the results are interpreted as elasticities at mean income the biases are as given in Corollary 3.2.[6]

Corollary 3.1: Biases in interpreting elasticities derived from formulas in Theorem 3 as mean elasticities (ME):

(i) The bias for the full price elasticity is

$$\phi_{3ij} - \phi_{1ij} = \frac{-\sum_{m=1}^{\infty}\left(\frac{\partial c_{mi}}{\partial \ln p_j}\right)h_m - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m} \quad .$$

(ii) The bias for the compensated price elasticity is

$$\eta_{3ij} - \eta_{1ij} = \frac{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mi} c_{nj}\left[h_m h_n - h_{m+n}\right] - \sum_{m=1}^{\infty}\left(\frac{\partial c_{mi}}{\partial \ln p_j}\right)h_m - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m+n-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} \quad .$$

(iii) The bias for the expenditure elasticity is

$$\varepsilon_{3i} - \varepsilon_{1i} = 0 \quad .$$

The estimator is thus unbiased for every income distribution.[7]

Corollary 3.1.1: For rank 2 and 3 $(R = 2,3)$ polynomials in logarithms of deflated expenditures defined in equation (2), biases in interpreting elasticies derived from formulas in Theorem 3 as mean elasticities (ME):

(i) The bias for the full price elasticity is

[6] Corollary 3.2 is in the spirit of the research dealing with the biases in using aggregate data to estimate micro structural price and income parameters (Blundell, Pashardes, and Weber, 1993, and Blundell, Meghir, and Weber, 1993), and biases in using aggregate macro based-elasticities to estimate micro elasticities at mean income (Denton and Mountain, 2001, 2004).

[7] Among the biases calculated, this is the only bias that is identically zero for all functional forms of demand systems.

$$\phi_{3ij} - \phi_{1ij} = \frac{-\sum_{r=1}^{R-1}\left(\frac{\partial a_{ri}}{\partial \ln p_j}\right)h_r - \sum_{r=1}^{R-1}\sum_{s=1}^{R-1} r a_{ri} a_{sj} h_{r-1} h_s}{\sum_{r=0}^{R-1} a_{ri} h_r} \quad . \quad ,$$

and thus is a function of GMIs up to order $R-1$.

(ii) The bias for the compensated price elasticity is

$$\eta_{3ij} - \eta_{1ij} = \frac{\sum_{r=1}^{R-1}\sum_{s=1}^{R-1} a_{ri} a_{sj}\left[h_r h_s - h_{r+s}\right] - \sum_{r=1}^{R-1}\left(\frac{\partial a_{ri}}{\partial \ln p_j}\right)h_r - \sum_{r=1}^{R-1}\sum_{s=1}^{R-1} r a_{ri} a_{sj} h_{r+s-1}}{\sum_{r=0}^{R-1} a_{ri} h_r} \quad .$$

and thus is a function of GMIs up to order $2(R-1)$.

(iii) The bias for the expenditure elasticity is

$$\varepsilon_{3i} - \varepsilon_{1i} = 0 \quad .$$

The estimator is thus unbiased for every income distribution.

Corollary 3.2: Biases in interpreting elasticies derived from formulas in Theorem 3 as elasticities at mean income (EM):

(i) The bias for the full price elasticity is

$$\phi_{3ij} - \phi_{2ij} = \frac{\frac{\partial c_{0i}}{\partial \ln p_j}\left(c_{0i} - \sum_{m=0}^{\infty} c_{mi} h_m\right) - c_{0i}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{c_{0i}\sum_{m=0}^{\infty} c_{mi} h_m} \quad .$$

(ii) The bias for the compensated price elasticity is

$$\eta_{3ij} - \eta_{2ij} = \frac{\left\{ c_{0i} \cdot \left[\frac{\partial c_{0i}}{\partial \ln p_j} + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nj} c_{mi} h_n h_m + \sum_{m=1}^{\infty} m c_{0j} c_{mi} h_{m-1} \right] - \left[\frac{\partial c_{0i}}{\partial \ln p_j} + c_{oj} c_{1i} + c_{0j} c_{0i} \right] \cdot \sum_{m=0}^{\infty} c_{mi} h_m \right\}}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad .$$

(iii) The bias for the expenditure elasticity is

$$\varepsilon_{3i} - \varepsilon_{2i} = \frac{c_{0i} \sum_{m=1}^{\infty} m c_{mi} h_{m-1} - c_{1i} \sum_{m=0}^{\infty} c_{mi} h_m}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m} \quad .$$

Corollary 3.2.1: For rank 2 and 3 $(R = 2,3)$polynomials in logarithms of deflated expenditures defined in equation (2), biases in interpreting elasticities derived from formulas in Theorem 3 as elasticities at mean income (EM):

(i) The bias for the full price elasticity is

$$\phi_{3ij} - \phi_{2ij} = \frac{\frac{\partial a_{0i}}{\partial \ln p_j} \left(a_{0i} - \sum_{r=0}^{R-1} a_{ri} h_r \right) - a_{0i} \sum_{r=1}^{R-1} \sum_{s=1}^{R-1} r a_{ri} a_{sj} h_{r-1} h_s}{a_{0i} \sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order $R-1$.

(ii) The bias for the compensated price elasticity is

$$\eta_{3ij} - \eta_{2ij} = \frac{\left\{ a_{0i} \cdot \left[\frac{\partial a_{0i}}{\partial \ln p_j} + \sum_{s=0}^{R-1} \sum_{r=0}^{R-1} a_{sj} a_{ri} h_s h_r + \sum_{r=1}^{R-1} r a_{0j} a_{ri} h_{r-1} \right] - \left[\frac{\partial a_{0i}}{\partial \ln p_j} + a_{oj} a_{1i} + a_{0j} a_{0i} \right] \cdot \sum_{r=0}^{R-1} a_{ri} h_r \right\}}{a_{0i} \sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order $R-1$.

(iii) The bias for the expenditure elasticity is

$$\varepsilon_{3i} - \varepsilon_{2i} = \frac{a_{0i} \sum_{r=1}^{R-1} r a_{ri} h_{r-1} - a_{1i} \sum_{r=0}^{R-1} a_{ri} h_r}{a_{0i} \sum_{r=0}^{R-1} a_{ri} h_r}$$

and thus is a function of GMIs up to order $R-1$.

All of the biases in Corollaries 3.1 and 3.2 are (in general) nonzero, with the exception of the expenditure elasticity bias in Corollary 3.1, where aggregate data are used to estimate the mean elasticity, and the bias is zero. The notion of a "representative consumer" is often invoked to justify the use of aggregate data. For the expenditure elasticity the representative consumer turns out in fact to be a household with mean elasticity, whatever the rank of the system and the distribution of income. For the price elasticities, though, that is not the case.

6. Illustrations

Four models of applied demand systems ranging from rank 2 to rank 4 that are familiar in the literature are the translog (TLOG), the linear Almost Ideal Demand System (AIDS), the quadratic extension of the linear system (QUAIDS), and Lewbel's rank 4 demand system, which we shall refer to as L4. TLOG and AIDS are rank 2 systems, QUAIDS is a rank 3 system. We use these four models to illustrate the biases discussed above.

The TLOG model (Christensen, Jorgenson, and Lau, 1975) is defined at the micro level by the equation[8]

$$w_{ik} = \frac{\alpha_i^* + \sum_{j=1}^{I} \gamma_{ij}^* \ln p_j - \sum_{j=1}^{I} \gamma_{ij}^* \ln x_k}{-1 + \sum_{i=1}^{I} \sum_{j=1}^{I} \gamma_{ij}^* \ln p_j} \quad . \qquad (11)$$

Under normalization of prices and income this becomes $w_{ik} = -\alpha_i^*$ and the corresponding aggregate form, consistent with equation (5), is $W_i = -\left(\alpha_i^* - \sum_{j=1}^{I} \gamma_{ij}^* h_1 \right)$.

The QUAIDS model (Banks, Blundell, and Lewbel, 1997) is defined by

$$w_{ik} = \alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i \ln(x_k / b) + \lambda_i (\ln(x_k / b))^2 / B \qquad (12)$$

with $\ln b = \sum_{i=1}^{I} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{I} \sum_{j=1}^{I} \gamma_{ij} (\ln p_i)(\ln p_j)$ and $\ln B = \sum_{i=1}^{I} \beta_i \ln p_i$

Under normalization this becomes $w_i = \alpha_i$, with corresponding aggregate form

$W_i = \alpha_i + \beta_i h_1 + \lambda_i h_2$.

The linear AIDS model (Deaton and Muellbauer, 1980) is obtained by setting $\lambda_i = 0, \forall i$, in equation (12), and omitting $\ln B$. W_i is then equal to $\alpha_i + \beta_i h_1$, under normalization.

The L4 model (Lewbel, 2003) is defined by

$$w_{ik} = \frac{\tau_i d}{x_k} + \left(1 - \frac{d}{x_k}\right)\left(\alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i [\ln(x_k - d)) - \ln b] + \lambda_i [\ln(x_k - d)) - \ln b]^2 / B \right) \qquad (13)$$

[8] This formulation of the translog model is also found in Jorgenson and Slesnick (1984).

with $\ln b = \alpha_0 + \sum_{i=1}^{I} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{I} \sum_{j=1}^{I} \gamma_{ij} (\ln p_i)(\ln p_j)$, $d = \rho_0 \prod_{i=1}^{I} p_i^{\tau_i}$ and $\ln B = \sum_{i=1}^{I} \beta_i \ln p_i$.[9] Note that L4 nests QUAIDS ($\rho_0 = 0$ *and* $\tau_i = 0$ *for* $i = 1,2,...,I$) and hence AIDS ($\lambda_i = 0$, additionally, for $i = 1,2,...,I$).

With normalization, equation (13) becomes $w_i = \tau_i \rho_0 + (1 - \rho_0)\alpha_i$.[10] In the form of equation (3), the Taylor series expansion of equation (13) is

$$w_{ik} = \sum_{m=0}^{3} c_{mi}(p)(\ln x_k)^m + \textit{terms of higher order in } m \quad (i = 1,2,...,I) \qquad (14)$$

with

$$c_{oi}(p) = \tau_i d + (1-d)\left(\alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i [\ln(1-d)) - \ln b] + \lambda_i [\ln(1-d)) - \ln b]^2 / B \right),$$

$$c_{1i}(p) = -\tau_i d + d \left(\alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i [\ln(1-d)) - \ln b] + \lambda_i [\ln(1-d)) - \ln b]^2 / B \right) + \beta_i + 2\frac{\lambda_i}{B}[\ln(1-d)) - \ln b],$$

$$c_{2i}(p) = \frac{1}{2}\left[\tau_i d - d \left(\alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i [\ln(1-d)) - \ln b] + \lambda_i [\ln(1-d)) - \ln b]^2 / B \right) + \frac{d}{1-d}\left(\beta_i + 2\frac{\lambda_i}{B}[\ln(1-d)) - \ln b] \right) + 2\frac{\lambda_i}{B(1-d)} \right],$$

[9] Two small typos appear in Lewbel's (2003) original paper. The corrected version of the model can be found in Lewbel (2004). The demand system in equation (13) is the correct version.

[10] Without loss of generality, part of the normalization for the L4 demand system is $\alpha_0 = \ln(1 - \rho_0)$.

$$c_{3i}(p) = \frac{1}{6}\left[-\tau_i d + d\left(\alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i [\ln(1-d)) - \ln b] + \lambda_i [\ln(1-d)) - \ln b]^2 / B \right) - \frac{d}{1-d}\left(1 + \frac{1}{1-d}\right)\left(\beta_i + 2\frac{\lambda_i}{B}[\ln(1-d) - \ln b]\right)\right];$$

and at $p_i = 1$ under normalization $c_{0i} = \tau_i \rho_0 + (1-\rho_0)\alpha_i$, $c_{1i} = -\tau_i \rho_0 + \rho_0 \alpha_i + \beta_i$,

$$c_{2i} = \frac{1}{2}\left[\tau_i \rho_o - \rho_0 \alpha_i + \frac{\rho_0}{1-\rho_0}\beta_i + \frac{2\lambda_i}{1-\rho_0}\right], c_{3i} = \frac{1}{6}\left[-\tau_i \rho_0 + \rho_0 \alpha_i - \frac{\rho_0}{1-\rho_0}\left(1 + \frac{1}{1-\rho_0}\right)\beta_i\right].$$

The formulas for the biases in the elasticities derived from these four models, corresponding to Corollaries 2.1.1, 3.1.1 and 3.2.1, are displayed in Tables 1, 2 and 3. (The biases in comparing AM and EM elasticities in Table 3 for AIDS can be found also in Denton and Mountain (2001) and for QUAIDS in Denton and Mountain (2004).) For convenience, we use again the following symbols in the tables: EM – elasticity at the mean of the income distribution; ME – mean elasticity based (correctly) on micro data; AM – elasticity at mean income based on aggregate data.

All of the biases shown in the tables are (in general) nonzero, with the notable exception of the expenditure elasticities in Table 2. Moreover, for the TLOG and AIDS models, as shown in Table 1, the EM expenditure elasticity is always greater than the ME expenditure elasticity (a positive bias) and as shown in Table 3 the AM expenditure elasticity is always less than the EM expenditure elasticity (a negative bias) for $h_1 > 0$.[11]

With respect to the TLOG and AIDS models with $h_1 > 0$ (as shown in Table 2) for full own-price elasticities and full cross-price elasticites, where both goods are luxuries ($\varepsilon_{2i}, \varepsilon_{2j} > 1$), or where both goods are necessities ($\varepsilon_{2i}, \varepsilon_{2j} < 1$), the bias in using an AM price elasticities to estimate am ME price elasticity is negative ($\phi_{3ij} < \phi_{1ij}$). On the other hand, for the TLOG and AIDS models with $h_1 > 0$, for full cross-price elasticites where one of the goods is a luxury and the other a necessity, the bias in using an AM cross-price elasticity to estimate an ME cross-price elasticity is positive

[11] For a wide range of income (expenditure) inequalities observed in OECD countries, calculations by Denton and Mountain (2001, 2007) show that h_1 is always positive.

($\phi_{3ij} > \phi_{1ij}$). In all of these situations, the larger is the expenditure inequality (the larger is h_1), the larger is the absolute value of the bias.

To give the results of these four demand systems some numerical perspective, we quantify the biases expressed theoretically in the Corollaries. We begin by assigning 'realistic' values to the micro expenditure and income distribution parameters. Values for the micro parameters are based on econometric estimates in Blundell, Pashardes, and Weber (1993). Under our normalization restrictions, we take mean w_i values (in rounded form) from table A1 of that paper for the six expenditure categories that the authors identify for estimation. (The seventh category was dropped by the authors because of the singularity of the expenditure system.) Values for the six micro expenditure and own-price compensated and full elasticities are based on the Blundell et al. generalized method of moments estimates in their tables 3A and 3B.[12] For the TLOG, AIDS and QUAIDS models, the calculation of micro parameters corresponding to the micro elasticities is straightforward. For the L4 model, the additional parameters ($\tau_i = 1,2,...,6$; ρ_0) must be chosen before calculation of the remaining ones. Because the ρ_0 parameter can be interpreted as a committed expenditure component (with $p_i = 1$, under normalization), we selected $\rho_0 = 0.2$ after consulting a number of related empirical estimates in the literature that use either the L4 model, or linear or quadratic expenditure models (e.g., Andrikopoulos, Brox, and Gamaletsos (1984), Howe, Pollak, and Wales (1979), Lewbel (2003), Lewis and Andrews (1989), Pollak and Wales (1978), Wales (1971)).

The values that we assign to the micro parameters are provided in our Appendix B, Table B1. We have retained, in that table and others, the names of the expenditure categories used by Blundell et al. (food, alcohol, fuel, clothing, transport, and services). However, we do that merely as a reminder that the parameter values we have chosen are 'realistic.' We emphasize that our calibrated model is not a model estimated by Blundell et al. We have simply used their results as a guide in calibrating our theoretical model.

For the income distribution parameters we assign values to h_1, h_2, h_3 and h_4 based on after-tax family income distributions reported in O'Higgins, Schmaus, and Stephenson

[12] Denton and Mountain (2004) used these same micro parameters for calculating biases in comparing AM and EM elasticities.

(1989, table 2). Values were calculated for seven OECD countries, reflecting a wide range of income distributions. The calculated values for h_1, h_2, h_3 *and* h_4 are provided in Table B2 of Appendix B. For the L4 model our estimates of biases are based on numerical approximations, where we made use of $h's$ up to h_4.

With the underlying micro parameters, we then calculated the biases reported in Tables 1, 2 and 3 for countries with the least and greatest inequality of income distribution, Sweden and Germany. Tables 4, 5, 6 and 7 report the corresponding mean elasticities (ME), the micro elasticity calculated at the mean of the income distribution (EM), and the elasticity at mean income based on aggregate data (AM) for all four models. Biases in estimating ME with either EM or AM elasticity are calculated. This is done for full own-price elasticities, compensated own-price elasticities and expenditure elasticities.

The results are similar across all four models. The main conclusions are as follows. Not unexpectedly, the greater the income inequality, the greater is the bias. (The biases for Sweden are generally smaller than those for Germany.) Furthermore, the greater the departure of the expenditure elasticity from one, the greater is the bias. For example, for food and alcohol, with EMs of 0.61 and 2.29, respectively, the biases are relatively large. This is in contrast with the results for clothing, with an EM of 0.92. Although not always, generally, in estimating the ME the AM does a better job than the EM. In terms of absolute size, the expenditure elasticity bias tends to be larger than the full price and compensated price elasticity biases.

For the L4 model we also tried different values of ρ_0 to investigate the sensitivity of the results to that parameter's value. When we changed the value of ρ_0 from 0.2 to 0.5, we found the biases in the price elasticities to be only slightly larger. However, we did find some much larger biases involving expenditure elasticities (e.g., $EM - ME$ biases are 0.113 and 0.235 for food in Sweden and Germany with $\rho_0 = 0.5$, compared with 0.055 and 0.111 for $\rho_0 = 0.2$).

7. Conclusion

We began this paper by noting that consumer-related policy decisions frequently require the evaluation of aggregate responses, often in the form of mean price or expenditure elasticities. Such elasticitities can be derived from a properly specified model fitted to micro data, but in practice that is seldom done. They can be approximated from a model fitted to aggregate data, but the approximation introduces the possibility of aggregation bias in the calculation of price elasticities, though interestingly not in the calculation of expenditure elasticities. We provide in this paper formulas for the correct calculation of mean elasticities – expenditure and both full and compensated price elasticities – and the corresponding biases when incorrect formulas are used. The correct formulas and the biases depend in general on the type of data (micro or aggregate), the type of model being estimated, the rank of the model, and the characteristics of the income distribution.

We have quantified the range of biases for familiar demand systems. The empirical results are robust in that the estimated biases are of the same order of magnitude, regardless of the functional form. Whether we use AM or EM elasticities to estimate the ME elasticity, the biases increase as the income inequality grows and as the underlying expenditure elasticities depart from one. Generally, the AM elasticity performs a better job than the EM elasticity in estimating ME.

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TABLE 1: BIASES IN USING EM TO ESTIMATE ME WITH SELECTED MODELS OF RANK 2, 3 OR 4

FULL PRICE ELASTICITY BIASES ($\phi_{2ij} - \phi_{1ij}$)

TLOG:

$$\left[-\gamma_{ij}^{*}\sum_{k}\gamma_{ik}^{*}h_{1}\right]\left[\alpha_{i}^{*}\left(\alpha_{i}^{*}-\sum_{k}\gamma_{ik}^{*}h_{1}\right)\right]^{-1}$$

AIDS:

$$\left[\left((\gamma_{ij}-\alpha_{j}\beta_{i})\right)\beta_{i}h_{1}\right]\left[\alpha_{i}(\alpha_{i}+\beta_{i}h_{1})\right]^{-1}$$

QUAIDS:

$$\left[(\gamma_{ij}-\alpha_{j}\beta_{i})(\alpha_{i}+\theta_{i})-\left(\gamma_{ij}-\alpha_{j}\left(\beta_{i}+2\lambda_{i}h_{1}\right)-\lambda_{i}\beta_{j}h_{2}\right)\alpha_{i}\right]\left[\alpha_{i}(\alpha_{i}+\theta_{i})\right]^{-1}$$

L4:

$$\frac{\chi_{0ij}\sum_{m=0}^{\infty}c_{mi}h_{m}-\left(\sum_{m=0}^{\infty}\chi_{mij}h_{m}\right)c_{0i}}{c_{0i}\sum_{m=0}^{\infty}c_{mi}h_{m}}$$ with c_{mi} defined by equation (14)

COMPENSATED PRICE ELASTICITY BIASES ($\eta_{2ij} - \eta_{1ij}$)

TLOG:

$$\left\{\begin{array}{l}\left[-\gamma_{ij}^{*}-\alpha_{i}^{*}\sum_{k}\gamma_{kj}^{*}-\alpha_{j}^{*}\sum_{k}\gamma_{ik}^{*}\right]\left[\sum_{k}\gamma_{ik}^{*}h_{1}\right]\\+\left[\alpha_{i}^{*}\left((-\alpha_{i}^{*}\sum_{k}\gamma_{kj}^{*}+2\sum_{k}\gamma_{ik}^{*}\sum_{k}\gamma_{kj}^{*})h_{1}+\sum_{k}\gamma_{ik}^{*}\sum_{k}\gamma_{kj}^{*}h_{2}\right)\right]\end{array}\right\}\left[\alpha_{i}^{*}\left(\alpha_{i}^{*}-\sum_{k}\gamma_{ik}^{*}h_{1}\right)\right]^{-1}$$

AIDS:

$$\left[(\gamma_{ij}+\alpha_{i}\alpha_{j})\beta_{i}h_{1}-\left((\alpha_{j}\beta_{i}+\alpha_{i}\beta_{j}+\beta_{i}\beta_{j})h_{1}+\beta_{i}\beta_{j}h_{2}\right)\alpha_{i}\right]\left[\alpha_{i}(\alpha_{i}+\beta_{i}h_{1})\right]^{-1}$$

QUAIDS:

$$\left[\begin{array}{l}(\gamma_{ij}+\alpha_{i}\alpha_{j})\theta_{i}-\alpha_{i}(\alpha_{j}\beta_{i}+\alpha_{i}\beta_{j}+\beta_{i}\beta_{j})h_{1}\\-\alpha_{i}((\alpha_{j}\lambda_{i}+\beta_{j}\beta_{i}+\lambda_{j}\beta_{i}+\beta_{j}\lambda_{i}+\alpha_{i}\lambda_{j})h_{2}+(\beta_{j}\lambda_{i}+\beta_{i}\lambda_{j}+2\lambda_{j}\lambda_{i})h_{3}+\lambda_{i}\lambda_{j}h_{4})\end{array}\right]\left[\alpha_{i}(\alpha_{i}+\theta_{i})\right]^{-1}$$

L4:

$$\left\{\left(\left(\chi_{0ij}+c_{0j}c_{0i}\right)+c_{oj}c_{1i}\right)\sum_{m=0}^{\infty}c_{mi}h_{m}-\left(\sum_{m=0}^{\infty}\chi_{mij}h_{m}+\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}mc_{mi}c_{nj}h_{m+n-1}+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}c_{mi}c_{nj}h_{m+n}\right)c_{0i}\right\}\left(c_{0i}\sum_{m=0}^{\infty}c_{mi}h_{m}\right)^{-1}$$

with c_{mi}, c_{nj} defined by equation (14)

EXPENDITURE ELASTICITY BIASES ($\varepsilon_{2i} - \varepsilon_{1i}$)

TLOG:

$$\left(\sum_k \gamma_{ik}^*\right)^2 h_1\left[\alpha_i^*\left(\alpha_i^* - \sum_k \gamma_{ik}^* h_1\right)\right]^{-1}$$

AIDS:

$$\beta_i^2 h_1\left[\alpha_i(\alpha_i + \beta_i h_1)\right]^{-1}$$

QUAIDS:

$$\left[\beta_i\left(\beta_i h_1 + \lambda_i h_2\right) - 2\alpha_i\lambda_i h_1\right]\left[\alpha_i(\alpha_i + \theta_i)\right]^{-1}$$

L4:

$$\frac{c_{1i}\sum_{m=0}^{\infty} c_{mi} h_m - c_{0i}\sum_{m=1}^{\infty} m c_{mi} h_{m-1}}{c_{0i}\sum_{m=0}^{\infty} c_{mi} h_m}$$ with c_{mi} defined by equation (14)

EM – elasticity at mean income; ME – mean elasticity; $\theta_i = \beta_i h_1 + \lambda_i h_2$; the first three values of χ_{im} are

$$\chi_{0ij} = \rho_9\tau_j\left(\tau_i - \alpha_i\right) + \left(1 - \rho_0\right)\left(\gamma_{ij} + \beta_i\frac{\left(\rho_0\left(\alpha_j - \tau_j\right) - \alpha_j\right)}{\left(1 - \rho_0\right)}\right);$$

$$\chi_{1ij} = \left(\alpha_i - \tau_j\right)\tau_j\rho_0 + \rho_0\left(\gamma_{ij} + \beta_i\frac{\left(\rho_0\left(\alpha_j - \tau_j\right) - \alpha_j\right)}{\left(1 - \rho_0\right)}\right) + 2\lambda_i\frac{\rho_0\left(\alpha_j - \tau_j\right) - \alpha_j}{\left(1 - \rho_0\right)};$$

$$\chi_{2ij} = \frac{1}{2}\left[\begin{array}{l}\tau_j\rho_0\left(\tau_i - \alpha_i\right) - \rho_0\left(\gamma_{ij} + \beta_i\frac{\left(\rho_0\left(\alpha_j - \tau_j\right) - \alpha_j\right)}{\left(1 - \rho_0\right)}\right) - 2\lambda_i\left(\frac{\beta_j}{\left(1 - \rho_0\right)} - \frac{\tau_j\rho_0}{\left(1 - \rho_0\right)^2}\right) + \\ \frac{\rho_0\tau_j\beta_i}{\left(1 - \rho_0\right)} + \rho_0\left(\frac{\rho_0\tau_j\beta_i}{\left(1 - \rho_0\right)^2} - 2\lambda_i\frac{\left(\tau_j\rho_0 + \alpha_j\right)}{\left(1 - \rho_0\right)}\right)\end{array}\right].$$

TABLE 2: BIASES IN USING AM TO ESTIMATE ME

FULL PRICE ELASTICITY BIASES ($\phi_{3ij} - \phi_{1ij}$)

TLOG:

$$\left[-\sum_k \gamma^*_{kj} \sum_k \gamma^*_{ik} h_1 \right]\left[-\alpha^*_i + \sum_k \gamma^*_{ik} h_1\right]^{-1}$$

AIDS:

$$\left[-\beta_i \beta_j h_1\right]\left[\alpha_i + \beta_i h_1\right]^{-1}$$

QUAIDS:

$$\left[\lambda_i \beta_j h_2 - (\beta_i + 2\lambda_i h_1)\theta_j\right]\left[\alpha_i + \theta_i\right]^{-1}$$

L4:

$$\frac{-\sum_{m=1}^{\infty} \kappa_{mij} h_m - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{\sum_{m=0}^{\infty} c_{mi} h_m} \quad \text{with } c_{mi}, c_{nj} \text{ defined by equation (14)}$$

COMPENSATED PRICE ELASTICITY BIASES ($\eta_{3ij} - \eta_{1ij}$)

TLOG:

$$\left[\sum_k \gamma^*_{ik} \sum_k \gamma^*_{kj} \left(h_1^{\ 2} - h_1 - h_2\right)\right]\left[-\alpha^*_i + \sum_k \gamma^*_{ik} h_1\right]^{-1}$$

AIDS:

$$\beta_i \beta_j \left[h_1^2 - h_1 - h_2\right]\left[\alpha_i + \beta_i h_1\right]^{-1}$$

QUAIDS:

$$\left[\beta_i \beta_j (h_1^2 - h_2 - h_1) + (\beta_i \lambda_j + \lambda_i \beta_j)(h_1 h_2 - h_3 - h_2) + \lambda_i \lambda_j (h_2^2 - h_4 - 2h_3)\right]\left[\alpha_i + \theta_i\right]^{-1}$$

L4:

$$\frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mi} c_{nj} \left[h_m h_n - h_{m+n}\right] - \sum_{m=1}^{\infty} \kappa_{mij} h_m - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m+n-1}}{\sum_{m=0}^{\infty} c_{mi} h_m}$$

with c_{mi}, c_{nj} defined by equation (14)

EXPENDITURE ELASTICITY BIASES ($\varepsilon_{3i} - \varepsilon_{1i}$)

TLOG:

No Bias

AIDS:

No Bias

QUAIDS:

No Bias

L4:

No Bias

AM – elasticity at mean income based on aggregate data; ME – mean elasticity; $\theta_\ell = \beta_\ell h_1 + \lambda_\ell h_2$ for $\ell = i, j;$ the first two values of χ_{im} are

$$\kappa_{1ij} = (\alpha_i - \tau_j)\tau_j \rho_0 + \rho_0 \left(\gamma_{ij} - \beta_i \frac{\rho_0 \tau_j}{(1-\rho_0)} \right) - 2\lambda_i \frac{\rho_0 \tau_j}{(1-\rho_0)};$$

$$\kappa_{2ij} = \frac{1}{2} \left[\begin{array}{l} \tau_j \rho_0 (\tau_i - \alpha_i) - \rho_0 \left(\gamma_{ij} - \beta_i \frac{\rho_0 \tau_j}{(1-\rho_0)} \right) - 2\lambda_i \left(\frac{\beta_j}{(1-\rho_0)} - \frac{\tau_j \rho_0}{(1-\rho_0)^2} \right) + \\ \frac{\rho_0 \tau_j \beta_i}{(1-\rho_0)} + \rho_0 \left(\frac{\rho_0 \tau_j \beta_i}{(1-\rho_0)^2} - 2\lambda_i \frac{\tau_j \rho_0}{(1-\rho_0)} \right) \end{array} \right].$$

TABLE 3: BIASES IN USING AM TO ESTIMATE EM WITH SELECTED MODELS OF RANK 2, 3 OR 4

FULL PRICE ELASTICITY BIASES ($\phi_{3ij} - \phi_{2ij}$)

TLOG:

$$\left[\left(\gamma_{ij}^* + \alpha_i^* \sum_k \gamma_{ik}^*\right) \sum_k \gamma_{ik}^* h_1\right]\left[\alpha_i^*\left(\alpha_i^* - \sum_k \gamma_{ik}^* h_1\right)\right]^{-1}$$

AIDS:

$$\left[\left(-\gamma_{ij} + (\alpha_j \beta_i - \alpha_i \beta_j)\right)\beta_i h_1\right]\left[\alpha_i (\alpha_i + \beta_i h_1)\right]^{-1}$$

QUAIDS:

$$\left[-\gamma_{ij}\theta_i - 2\lambda_i \alpha_i (\alpha_j + \theta_j) + \beta_i (\alpha_j \theta_i - \alpha_i \theta_j)\right]\left[\alpha_i (\alpha_i + \theta_i\ )\right]^{-1}$$

L4:

$$\frac{\chi_{0ij}\left(c_{0i} - \sum_{m=0}^{\infty} c_{mi} h_m\right) - c_{0i} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m c_{mi} c_{nj} h_{m-1} h_n}{c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m}$$

with c_{mi}, c_{nj} defined by equation (14)

COMPENSATED PRICE ELASTICITY BIASES ($\eta_{3ij} - \eta_{2ij}$)

TLOG:

$$\left[\left(\gamma_{ij}^* + \alpha_j^* \sum_k \gamma_{ik}^*\right) \sum_k \gamma_{ik}^* h_1\right]\left[\alpha_i^*\left(\alpha_i^* - \sum_k \gamma_{ik}^* h_1\right)\right]^{-1} + \sum_k \gamma_{kj}^* h_1$$

AIDS:

$$-\gamma_{ij}\beta_i h_1\left[\alpha_i (\alpha_i + \beta_i h_1)\right]^{-1} + \beta_j h_1$$

QUAIDS:

$$-\gamma_{ij}\theta_i\left[\alpha_i (\alpha_i + \theta_i)\right]^{-1} + \theta_j$$

L4:

$$\left\{c_{0i} \cdot \left[\chi_{0ij} + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nj} c_{mi} h_n h_m + \sum_{m=1}^{\infty} m c_{0j} c_{mi} h_{m-1}\right] - \left[\chi_{0ij} + c_{oj} c_{1i} + c_{0j} c_{0i}\right] \cdot \sum_{m=0}^{\infty} c_{mi} h_m\right\}\left(c_{0i} \sum_{m=0}^{\infty} c_{mi} h_m\right)^{-1}$$

with c_{mi}, c_{nj} defined by equation (14)

EXPENDITURE ELASTICITY BIASES ($\varepsilon_{3i} - \varepsilon_{2i}$)

TLOG:

$$-\left(\sum_k \gamma_{ik}^*\right)^2 h_1\left[\alpha_i^*\left(\alpha_i^* - \sum_k \gamma_{ik}^* h_1\right)\right]^{-1}$$

AIDS:

$$-\beta_i{}^2 h_1\left[\alpha_i(\alpha_i + \beta_i h_1)\right]^{-1}$$

QUAIDS:

$$\left(2\alpha_i \lambda_i h_1 - \beta_i \theta_i\right)\left[\alpha_i(\alpha_i + \theta_i)\right]^{-1}$$

L4:

$$\frac{c_{0i}\sum_{m=1}^{\infty} m c_{mi} h_{m-1} - c_{1i}\sum_{m=0}^{\infty} c_{mi} h_m}{c_{0i}\sum_{m=0}^{\infty} c_{mi} h_m}$$ with c_{mi} defined by equation (14)

AM – elasticity at mean income based on aggregate data; EM – elasticity at the mean income; $\theta_\ell = \beta_\ell h_1 + \lambda_\ell h_2$ for $\ell = i, j$; $\chi_{0ij} = \rho_9 \tau_j(\tau_i - \alpha_i) + (1-\rho_0)\left(\gamma_{ij} + \beta_i \frac{\left(\rho_0(\alpha_j - \tau_j) - \alpha_j\right)}{(1-\rho_0)}\right)$

TABLE 4: ELASTICITY BIASES FOR TLOG DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.535	-0.521	-1.627	-1.551	-0.507	-0.498	-0.618	-0.615	-0.702	-0.708	-0.736	-0.745
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.542	-0.507	-1.639	-1.571	-0.508	-0.499	-0.618	-0.615	-0.703	-0.709	-0.738	-0.750
EM – ME	-0.029	-0.043	-0.113	-0.189	-0.010	-0.019	-0.004	-0.007	0.006	0.012	0.012	0.021
AM - ME	-0.007	-0.014	-0.012	-0.020	-0.001	-0.001	-0.000	-0.000	-0.001	-0.001	-0.002	-0.005
Compensated Own-Price												
ME	-0.325	-0.301	-1.432	-1.334	-0.441	-0.433	-0.527	-0.524	-0.480	-0.481	-0.550	-0.549
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.345	-0.338	-1.468	-1.391	-0.442	-0.435	-0.527	-0.525	-0.483	-0.485	-0.558	-0.563
EM – ME	-0.025	-0.049	-0.148	-0.246	-0.009	-0.017	-0.003	-0.006	0.000	0.001	-0.000	-0.001
AM - ME	-0.020	-0.037	-0.036	-0.057	-0.001	-0.002	-0.000	-0.001	-0.003	-0.004	-0.008	-0.014
Expenditure												
ME	0.590	0.572	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.590	0.572	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM – ME	0.020	0.038	0.177	0.294	0.003	0.006	0.001	0.001	0.005	0.009	0.024	0.042
AM - ME	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 5: ELASTICITY BIASES FOR AIDS DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.542	-0.521	-1.639	-1.571	-0.508	-0.499	-0.618	-0.615	-0.703	-0.709	-0.738	-0.750
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.548	-0.534	-1.651	-1.592	-0.508	-0.499	-0.618	-0.615	-0.704	-0.711	-0.741	-0.755
EM – ME	-0.022	-0.043	-0.101	-0.169	-0.009	-0.018	-0.004	-0.007	-0.007	0.013	0.014	0.026
AM - ME	-0.006	-0.013	-0.012	-0.021	-0.000	-0.000	-0.000	-0.000	-0.001	-0.002	-0.003	-0.005
Compensated Own-Price												
ME	-0.332	-0.315	-1.445	-1.354	-0.441	-0.434	-0.527	-0.525	-0.481	-0.482	-0.553	-0.555
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.352	-0.352	-1.480	-1.411	-0.442	-0.435	-0.527	-0.525	-0.484	-0.487	-0.561	-0.568
EM – ME	-0.018	-0.035	-0.135	-0.226	-0.009	-0.016	-0.003	-0.005	0.001	0.002	0.003	0.005
AM - ME	-0.020	-0.037	-0.035	-0.057	-0.001	-0.001	-0.000	-0.000	-0.003	-0.005	-0.008	-0.013
Expenditure												
ME	0.590	0.572	2.113	1.996	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.590	0.590	2.113	2.113	0.837	0.834	0.919	0.919	1.195	1.191	1.426	1.408
EM – ME	0.020	0.038	0.177	0.294	0.003	0.006	0.001	0.001	0.005	0.009	0.024	0.042
AM - ME	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 6: ELASTICITY BIASES FOR QUAIDS DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.538	-0.513	-1.642	-1.576	-0.565	-0.601	-0.586	-0.549	-0.713	-0.727	-0.715	-0.706
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.545	-0.527	-1.654	-1.596	-0.566	-0.604	-0.587	-0.551	-0.714	-0.729	-0.718	-0.712
EM – ME	-0.026	-0.051	-0.098	-0.164	0.048	0.084	-0.036	-0.073	0.017	0.031	-0.009	-0.018
AM - ME	-0.007	-0.014	-0.012	-0.020	-0.001	-0.003	-0.001	-0.002	-0.001	-0.002	-0.003	-0.006
Compensated Own-Price												
ME	-0.331	-0.311	-1.449	-1.361	-0.483	-0.503	-0.506	-0.479	-0.483	-0.484	-0.547	-0.543
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.352	-0.352	-1.484	-1.417	-0.482	-0.506	-0.508	-0.485	-0.487	-0.491	-0.550	-0.550
EM – ME	-0.019	-0.039	-0.131	-0.219	0.033	0.053	-0.024	-0.051	0.003	0.004	-0.003	-0.007
AM - ME	-0.021	-0.041	-0.035	-0.056	0.001	-0.003	-0.002	-0.006	-0.004	-0.007	-0.003	-0.007
Expenditure												
ME	0.582	0.555	2.114	1.996	0.958	1.044	0.845	0.769	1.211	1.220	1.394	1.347
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.582	0.555	2.114	1.996	0.958	1.044	0.845	0.769	1.211	1.220	1.394	1.347
EM – ME	0.028	0.055	0.176	0.294	-0.118	-0.204	0.075	0.151	-0.011	-0.020	0.056	0.103
AM - ME	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 7: ELASTICITY BIASES FOR L4 DEMAND SYSTEM

	Food		Alcohol		Fuel		Clothing		Transport		Services	
	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany	Sweden	Germany
Full Own-Price												
ME	-0.532	-0.496	-1.612	-1.534	-0.558	-0.588	-0.578	-0.536	-0.717	-0.733	-0.714	-0.711
EM	-0.564	-0.564	-1.740	-1.740	-0.517	-0.517	-0.622	-0.622	-0.696	-0.696	-0.724	-0.724
AM	-0.536	-0.505	-1.629	-1.563	-0.557	-0.588	-0.578	-0.537	-0.719	-0.738	-0.718	-0.718
EM – ME	-0.032	-0.068	-0.128	-0.206	0.041	0.071	-0.044	-0.086	0.021	0.037	-0.010	-0.013
AM - ME	-0.004	-0.009	-0.017	-0.029	0.001	0.000	-0.000	-0.001	-0.002	-0.005	-0.004	-0.007
Compensated Own-Price												
ME	-0.332	-0.306	-1.406	-1.296	-0.478	-0.494	-0.501	-0.469	-0.480	-0.478	-0.543	-0.542
EM	-0.350	-0.350	-1.580	-1.580	-0.450	-0.450	-0.530	-0.530	-0.480	-0.480	-0.550	-0.550
AM	-0.356	-0.354	-1.451	-1.370	-0.476	-0.495	-0.503	-0.476	-0.487	-0.491	-0.548	-0.552
EM – ME	-0.018	-0.044	-0.174	-0.284	0.028	0.044	-0.029	-0.061	0.000	-0.002	-0.007	-0.008
AM - ME	-0.024	-0.048	-0.045	-0.074	0.002	-0.001	-0.002	-0.007	-0.007	-0.013	-0.005	-0.010
Expenditure												
ME	0.555	0.499	2.110	1.996	0.944	1.023	0.827	0.732	1.221	1.238	1.400	1.358
EM	0.610	0.610	2.290	2.290	0.840	0.840	0.920	0.920	1.200	1.200	1.450	1.450
AM	0.555	0.499	2.110	1.996	0.958	1.023	0.827	0.732	1.221	1.258	1.400	1.358
EM – ME	0.055	0.111	0.180	0.294	-0.104	-0.183	0.093	0.188	-0.021	-0.038	0.050	0.092
AM - ME	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Appendix A – Theorem and Corollary Proofs

Proof of Theorem 1:

(i) Using equations (3) and (4) (and using the normalization restrictions, both here and subsequently), the *mean full price elasticity* (ϕ_{1ij}) is given by

$$\begin{aligned}
\phi_{1ij} &= \sum_{k=1}^{K} \frac{\partial \ln q_{ik}}{\partial \ln p_j} \frac{q_{ik}}{Q_i} \\
&= \sum_{k=1}^{K} \left(-\delta_{ij} + \frac{\dfrac{\partial w_{ik}}{\partial \ln p_j}}{w_{ik}} \right) \frac{q_{ik}}{Q_i} \\
&= \sum_{k=1}^{K} \left(-\delta_{ij} + \frac{\sum_{m=0}^{\infty} \left[\dfrac{\partial c_{mi}}{\partial \ln p_j} \right] (\ln x_k)^m}{w_{ik}} \right) \frac{q_{ik}}{Q_i} \\
&= -\delta_{ij} + \sum_{k=1}^{K} \left(\left(\frac{\sum_{m=0}^{\infty} \left[\dfrac{\partial c_{mi}}{\partial \ln p_j} \right] (\ln x_k)^m}{W_i} \right) \frac{x_k}{X} \right) \\
&= -\delta_{ij} + \frac{\sum_{m=0}^{\infty} \left[\dfrac{\partial c_{mi}}{\partial \ln p_j} \right] h_m}{\sum_{m=0}^{\infty} c_{mi} h_m} \qquad (A1)
\end{aligned}$$

(ii) With respect to the second term of equation (7), using equations (3) and (4) again,

$$\sum_{k=1}^{K} w_{jk} \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i} = \sum_{k=1}^{K} w_{jk} \left[\frac{\partial \ln\left(\frac{w_{ik} x_k}{p_i}\right)}{\partial \ln x_k} \frac{w_{ik}}{W_i} \frac{x_k}{X} \right]$$

$$= \sum_{k=1}^{K} w_{jk} \left[\left(1 + \frac{\sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1}}{w_{ik}} \right) \frac{w_{ik}}{W_i} \frac{x_k}{X} \right]$$

$$= \sum_{k=1}^{K} \frac{w_{jk} w_{ik}}{W_i} \frac{x_k}{X} + \frac{\sum_{k=1}^{K} w_{jk} \sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1} \frac{x_k}{X}}{W_i}$$

$$= \frac{\sum_{k=1}^{K} \sum_{n=0}^{\infty} c_{nj} (\ln x_k)^n \sum_{m=0}^{\infty} c_{mi} (\ln x_k)^m \frac{x_k}{X} + \sum_{k=1}^{K} \sum_{n=0}^{\infty} c_{nj} (\ln x_k)^n \sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1} \frac{x_k}{X}}{W_i}$$

$$= \frac{\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mi} c_{nj} h_{m+n} + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} m c_{mi} c_{nj} h_{m+n-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} . \qquad (A2)$$

The *mean compensated price elasticity* (η_{1ij}) of Theorem 1 is then derived by adding (A2) to (A1).

(iii) Using equations (2), (3) and (5), the *mean expenditure elasticity* is

$$\varepsilon_{1i} = \sum_{k=1}^{K} \frac{\partial \ln q_{ik}}{\partial \ln x_k} \cdot \frac{q_{ik}}{Q_i}$$

$$= \sum_{k=1}^{K} \left[\left(1 + \frac{\sum_{m=1}^{\infty} m c_{mi} (\ln x_k)^{m-1}}{w_{ik}} \right) \frac{w_{ik}}{W_i} \frac{x_k}{X} \right]$$

$$= 1 + \frac{\sum_{m=1,}^{\infty} m c_{mi} \sum_{k=1}^{K} \left(\frac{x_k}{X} \right) (\ln x_k)^{m-1}}{W_i}$$

$$= 1 + \frac{\sum_{m=1,}^{\infty} m c_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} \qquad (A3)$$

Proof of Theorem 2:

The *full price elasticity evaluated at mean income* and the *expenditure elasticity evaluated at mean income* can be derived directly from equation (2). The *compensated price elasticity evaluated at mean income* then follows from equation (4).

Proof of Corollary 2.1:

The biases follow directly from computing $\phi_{2ij}-\phi_{1ij}$, $\eta_{2ij}-\eta_{1ij}$, and $\varepsilon_{2i}-\varepsilon_{1i}$, where ϕ_{1ij}, η_{1ij}, and ε_{1i} are as calculated in Theorem 1 and ϕ_{2ij}, η_{2ij}, and ε_{2i} are as calculated in Theorem 2.

Proof of Theorem 3:

(i) Using equations (9) and (4), and taking $\frac{\partial \hat{c}_{0i}(p)}{\partial \ln p_j}$ to be the same as $\frac{\partial c_{0i}(p)}{\partial \ln p_j}-\hat{c}_{1i}\sum_{n=1}^{\infty} c_{nj}h_n$ (the conventional assumption in working with a model based on aggregate data), the *full price elasticity* is

$$
\begin{aligned}
\phi_{3ij} &= \frac{\partial \ln Q_i}{\partial \ln p_j} \\
&= -\delta_{ij}+\frac{\dfrac{\partial \hat{c}_{0i}}{\partial \ln p_j}}{W_i} \\
&= -\delta_{ij}+\frac{\dfrac{\partial c_{0i}(p)}{\partial \ln p_j}-\hat{c}_{1i}\sum_{n=1}^{\infty} c_{nj}h_n}{\sum_{m=0}^{\infty} c_{mi}h_m} \\
&= -\delta_{ij}+\frac{\dfrac{\partial c_{0i}(p)}{\partial \ln p_j}-\sum_{m=1}^{\infty} mc_{mi}h_{m-1}\sum_{n=1}^{\infty} c_{nj}h_n}{\sum_{m=0}^{\infty} c_{mi}h_m}, \qquad (A4)
\end{aligned}
$$

(ii) The second component of equation (7), again using equations (6) and (3), can be written as

$$W_j \frac{\partial \ln Q_i}{\partial \ln X} = \frac{W_j}{W_i}\left(W_i + \sum_{m=1}^{\infty} mc_{mi} h_{m-1}\right)$$

$$= \frac{\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nj} c_{mi} h_n h_m + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} mc_{nj} c_{mi} h_n h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} \qquad (A5)$$

The *compensated price elasticity* (η_{3ij}) is then obtained by adding (A5) to (A4).

(iii) The *expenditure elasticity evaluated at its mean*, in Theorem 3 is

$$\varepsilon_{3i} = \frac{\partial \ln Q_i}{\partial \ln X}$$

$$= 1 + \frac{\sum_{m=1}^{\infty} mc_{mi} h_{m-1}}{\sum_{m=0}^{\infty} c_{mi} h_m} \qquad (A6).$$

Proof of Corollary 3.1:

The biases follow directly from computing $\phi_{3ij} - \phi_{1ij}$, $\eta_{3ij} - \eta_{1ij}$, and $\varepsilon_{3i} - \varepsilon_{1i}$, where ϕ_{1ij}, η_{1ij}, and ε_{1i} are as calculated in Theorem 1 and ϕ_{3ij}, η_{3ij}, and ε_{3i} are as calculated in Theorem 3.

Proof of Corollary 3.2:

The biases follow directly from computing $\phi_{3ij} - \phi_{2ij}$, $\eta_{3ij} - \eta_{2ij}$, and $\varepsilon_{3i} - \varepsilon_{2i}$, where ϕ_{2ij}, η_{2ij}, and ε_{2i} are as calculated in Theorem 2 and ϕ_{3ij}, η_{3ij}, and ε_{3i} are as calculated in Theorem 3.

APPENDIX B: PARAMETER VALUES USED IN CALIBRATION

TABLE B1: UNDERLYING EM ELASTICITIES AND PARAMETERS OF MICRO MODEL

	FOOD	ALCOHOL	FUEL	CLOTHING	TRANSPORT	SERVICES
EM						
ϕ_{ii}	-0.564	-1.740	-0.517	-0.622	-0.696	-0.724
η_{ii}	-0.350	-1.580	-0.450	-0.530	-0.480	-0.550
ε_i	0.610	2.290	0.840	0.920	1.200	1.450
TLOG						
α_i^*	-0.35	-0.07	-0.08	-0.10	-0.18	-0.12
γ_{ij}^*	-0.2005	0.0581	-0.0396	-0.0386	-0.0482	-0.0266
$\sum_j \gamma_{ij}^*$	-0.1365	0.0903	-0.0128	-0.008	0.036	0.054
AIDS						
α_i	0.35	0.07	0.08	0.10	0.18	0.12
γ_{ii}	0.105	-0.0455	0.0376	0.037	0.0612	0.0396
β_i	-0.1365	0.0903	-0.0128	-0.008	0.036	0.054
QUAIDS						
α_i	0.35	0.07	0.08	0.10	0.18	0.12
γ_{ii}	0.105	-0.0455	0.0376	0.037	0.0612	0.0396
β_i	-0.1365	0.0903	-0.0128	-0.008	0.036	0.054
λ_i	-0.008	-0.002	0.037	-0.026	0.015	-0.027
L4						
$\rho_0 = .2$						
α_i	0.3325	0.0825	0.0475	0.105	0.185	0.1325
γ_{ii}	0.1297	-0.0577	0.0417	0.0461	0.0764	0.0487
β_i	-0.119	0.0778	0.0197	-0.013	0.031	0.0415
λ_i	-0.008	-0.002	0.037	-0.026	0.015	-0.027
τ_i	0.42	0.02	0.21	0.08	0.16	0.07

TABLE B2: INCOME DISTRIBUTION PARAMETERS FOR SEVEN COUNTRIES

	h_1	h_2	h_3	h_4
Sweden	.123	.242	.015	.113
Norway	.142	.259	-.023	.168
Israel	.166	.313	.006	.219
Canada	.171	.321	-.007	.263
United Kingdom	.172	.321	-.002	.232
United States	.204	.367	-.016	.345
Germany	.229	.456	.105	.412

NOTE: Calculations are based on after-tax family income quintile shares provided in table 2 of O'Higgins, Schmaus, and Stephenson (1989). See Denton and Mountain (2004) for details of the calculations.

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